OKINAWA SCHOOL IN PHYSICS: COHERENT QUANTUM DYNAMICS
Sep.28th-Oct. 9th @OIST

Interdisciplinary fields between quantum information science and physics

Keisuke Fujii
The Hakubi center for advanced research/
Graduate School of Science
Kyoto University

Hakubi

京都大学
KYOTO UNIVERSITY

Outline of 3 Days

Lecture 1: foundations of quantum computation

-elementary gates and universal quantum computation

-quantum algorithms

-quantum stabilizer formalism (graph state, quantum error correction)

Lecture 2: 1D quantum system

-what is quantum phase

-how it is useful for QIP

Lecture 3: 2D quantum system

-topologically ordered system

-how it is related to quantum error correction codes

-how topologically protected quantum computation works

Today's topic

Topologically ordered system in 2D

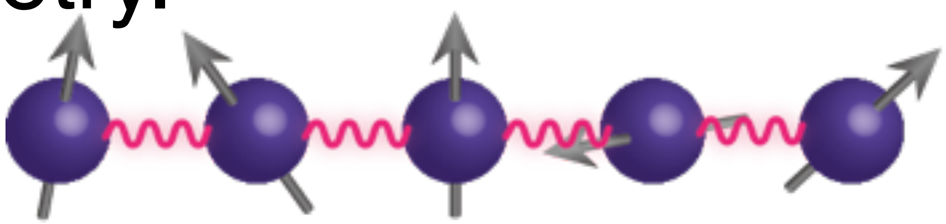
How it is useful for QIP

How topologically protected quantum information is implemented

Keywords: Kitaev's toric code, logical operators, topological quantum error correction

In yesterday's talk

Unpaired Majorana fermion is protected by symmetry.



What if your system has no symmetry, such as the fermionic parity preservation?

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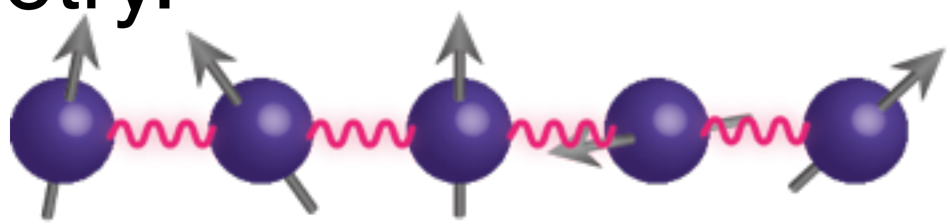

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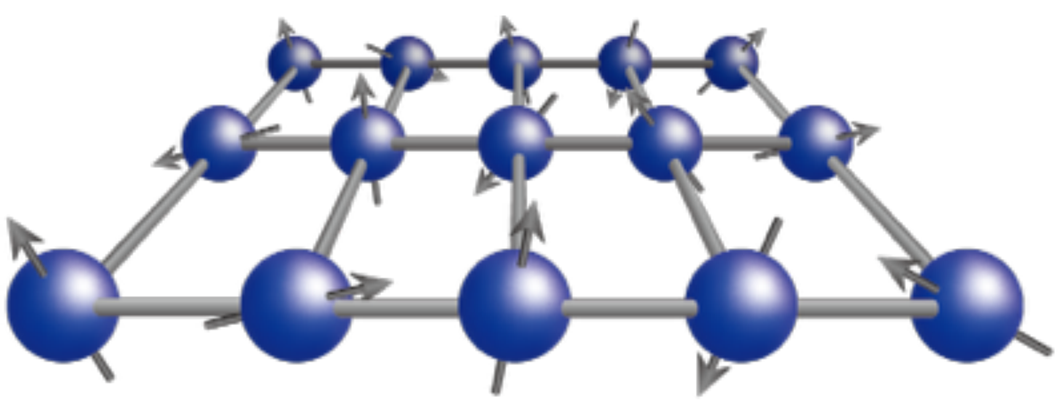
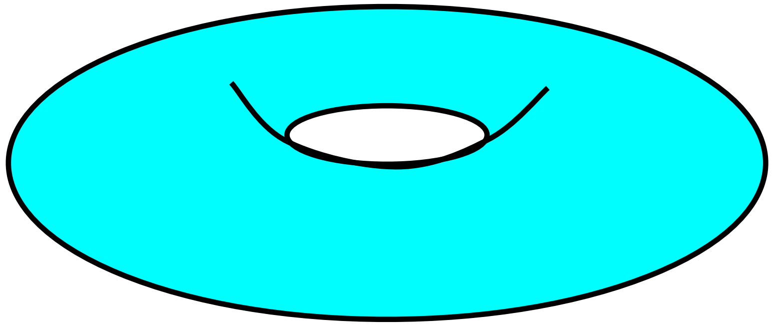




intrinsic topological order in D≧2

Recall: stabilizer codes

Pauli group: $\mathcal{P}_n = \{\pm 1, \pm i\} \times \{I, X, Y, Z\}^{\otimes n}$

Stabilizer group: $\mathcal{S}_n \subset \mathcal{P}_n$

$$[S_i, S_j] = 0, \quad S_i^\dagger = S_i \text{ for all } S_i, S_j \in \mathcal{S}_n$$

Hermitian Abelian subgroup

stabilizer group is specified by the set of generators $\langle\{S_i\}\rangle$.

Stabilizer code states:

$$|\Psi\rangle = S_i|\Psi\rangle \quad \text{for all } S_i \in \mathcal{S}_n$$

Logical operators: commute with / independent of the stabilizer group

ex) $\mathcal{S} = \langle ZZI, IZZ\rangle, \quad L_X = XXX, L_Z = IIZ$

→ $\{|000\rangle, |111\rangle\}$ $\qquad IIZ|111\rangle = -|111\rangle, \quad XXX|000\rangle = |111\rangle$

logical operators act nontrivially inside the code space

Kitaev's toric code[Kitaev97]

Kitaev, Annals Phys. **303**, 2 (2003)

Stabilizer operators:

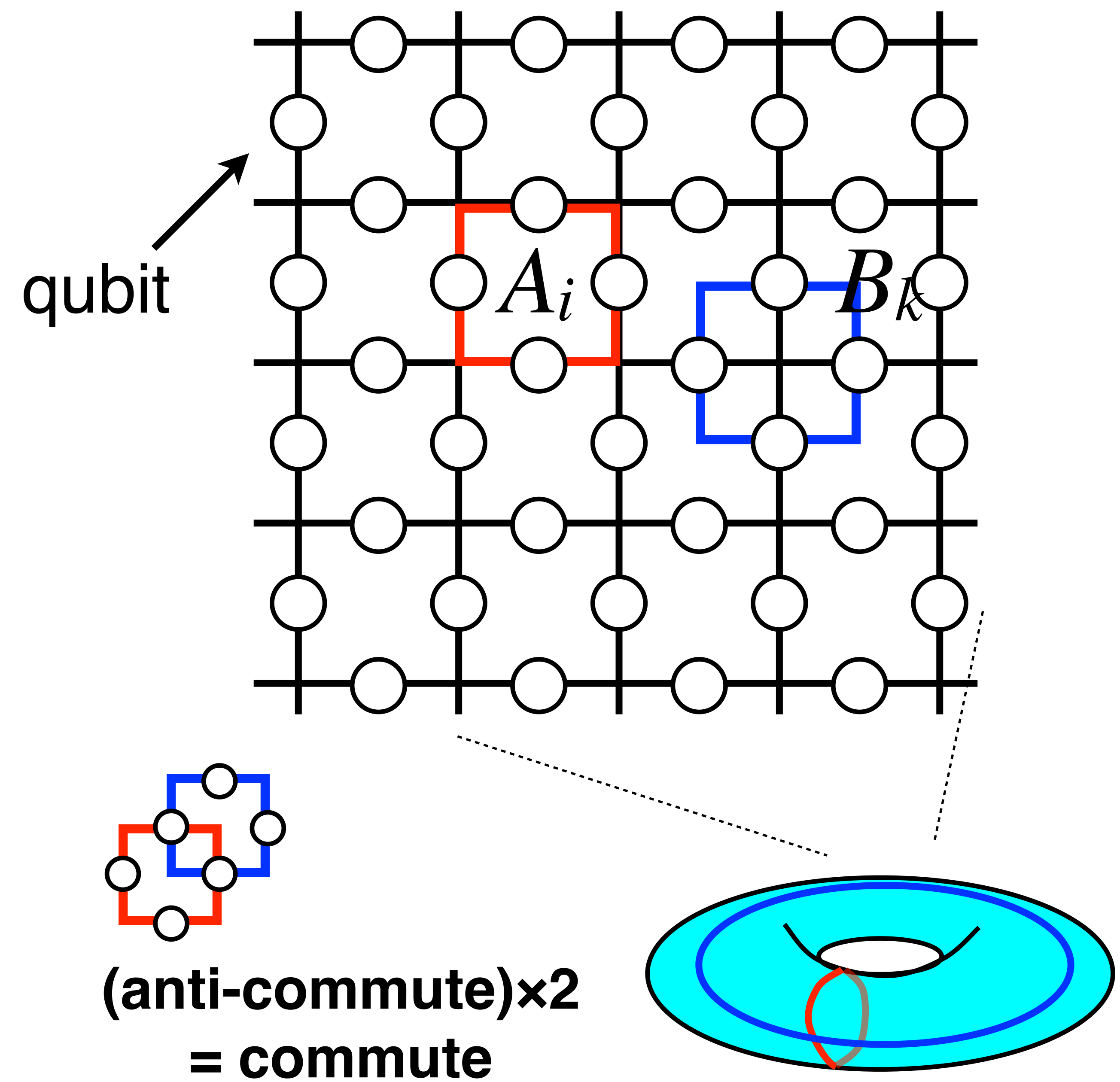
face (plaquette) operator:

$$A_i = \prod_{l \in \partial f_i} Z_l = Z(\partial f_i),$$

face (plaquette) operator:

$$B_k = \prod_{\bar{l} \in \partial \bar{f}_k} X_{\bar{l}} = X(\partial \bar{f}_k).$$

These are all commutable.



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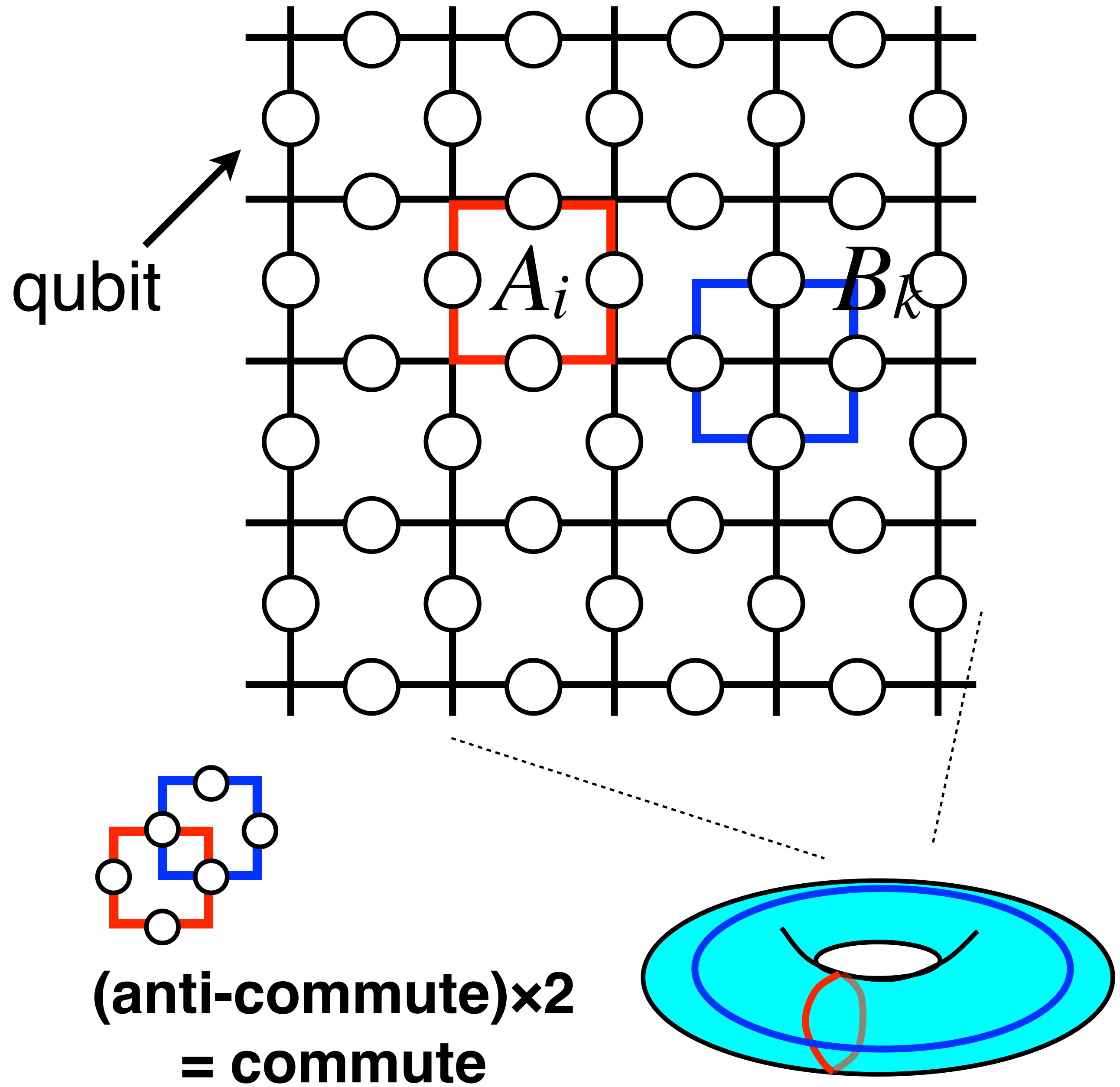


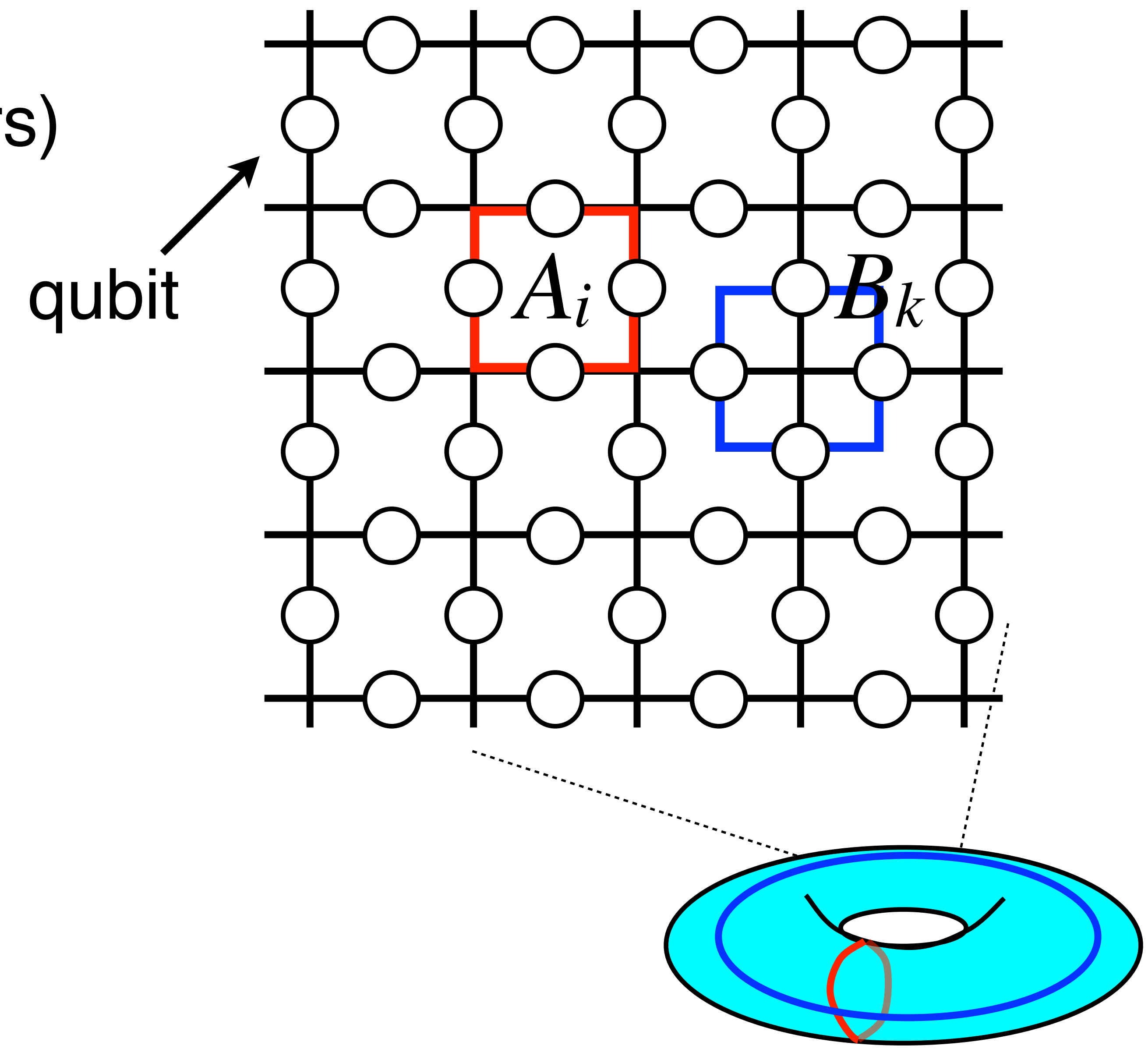
These are all commutable.

For all faces and vertices,

$$A_i|\Psi\rangle = |\Psi\rangle, \quad B_k|\Psi\rangle = |\Psi\rangle$$

Degeneracy of toric code

dim of stabilizer subspace
= 2^(# of qubits - # of generators)



Degeneracy of toric code

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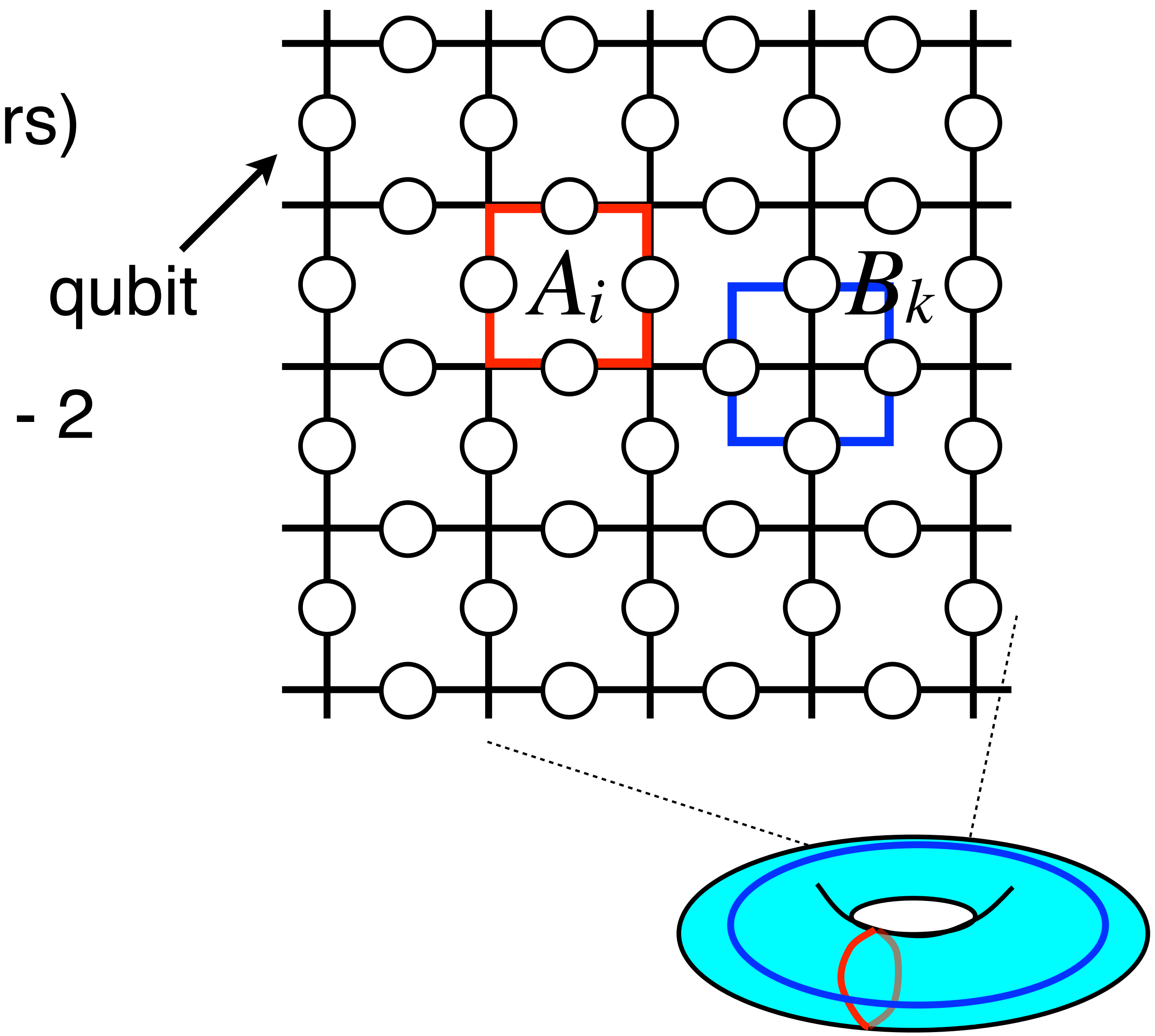
#qubit = $|E|$ on $N \times N$ torus → $2N^2$
#generator = $(|F| + |V| - 2)$ → $2N^2 - 2$



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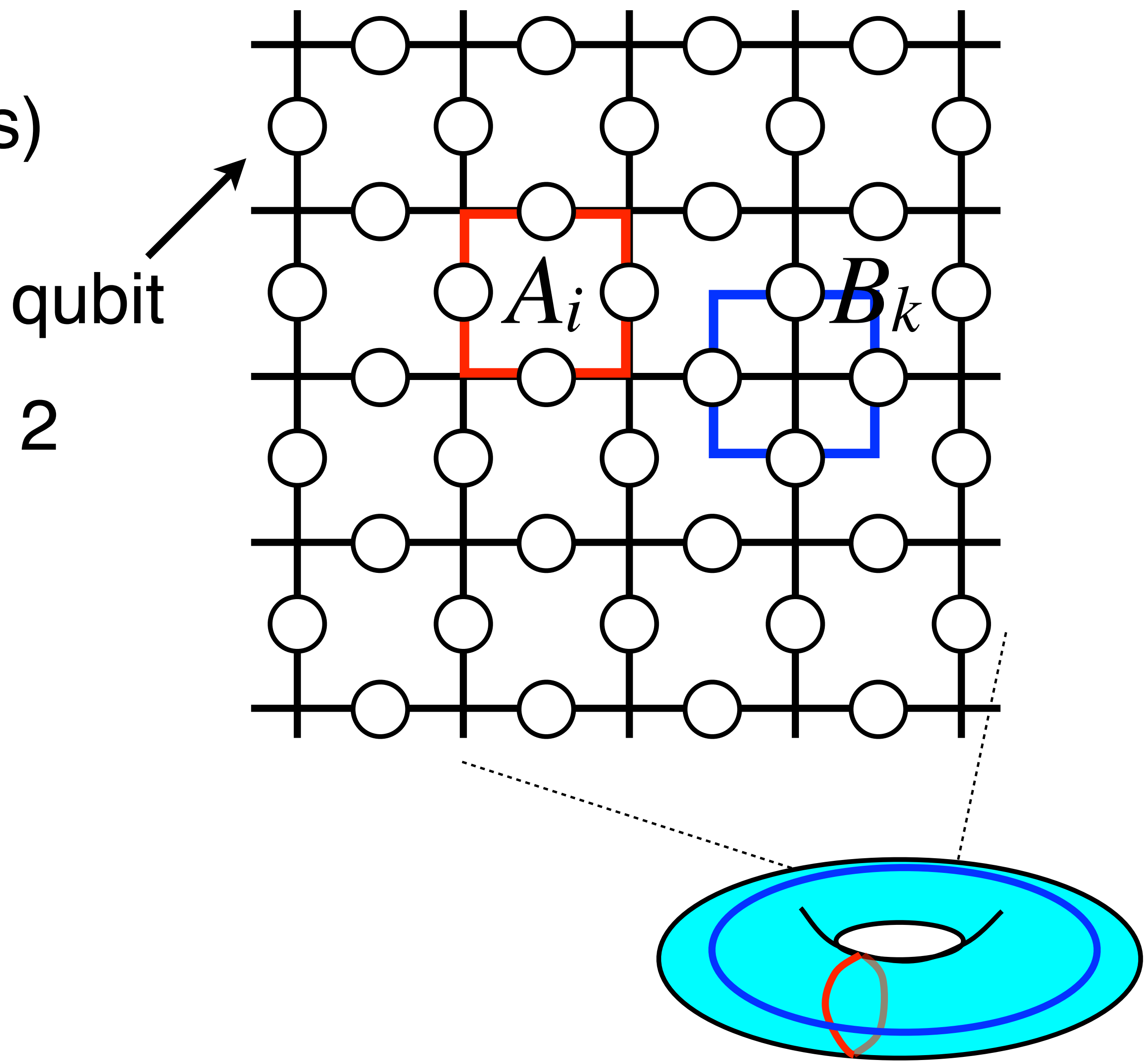

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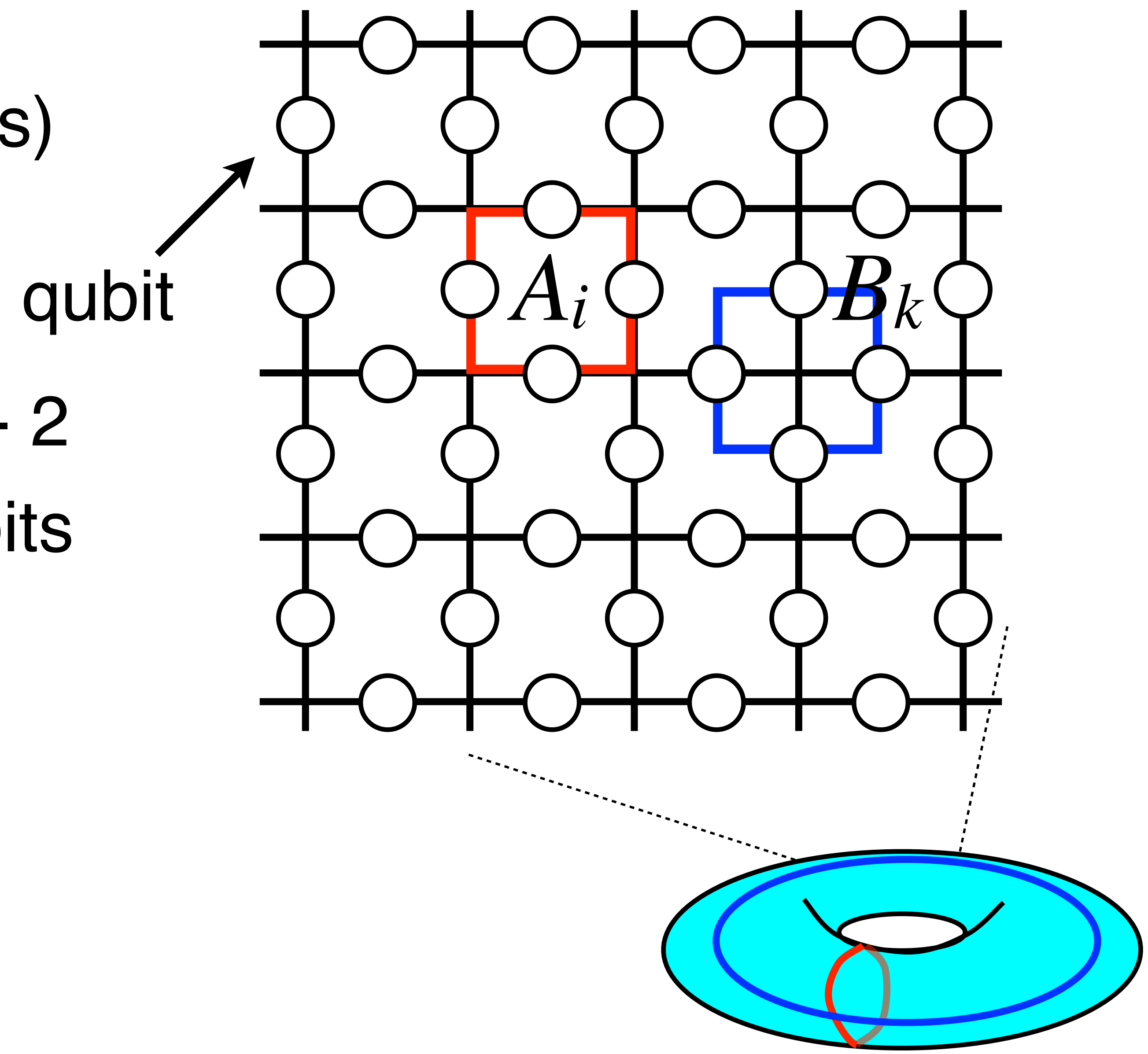

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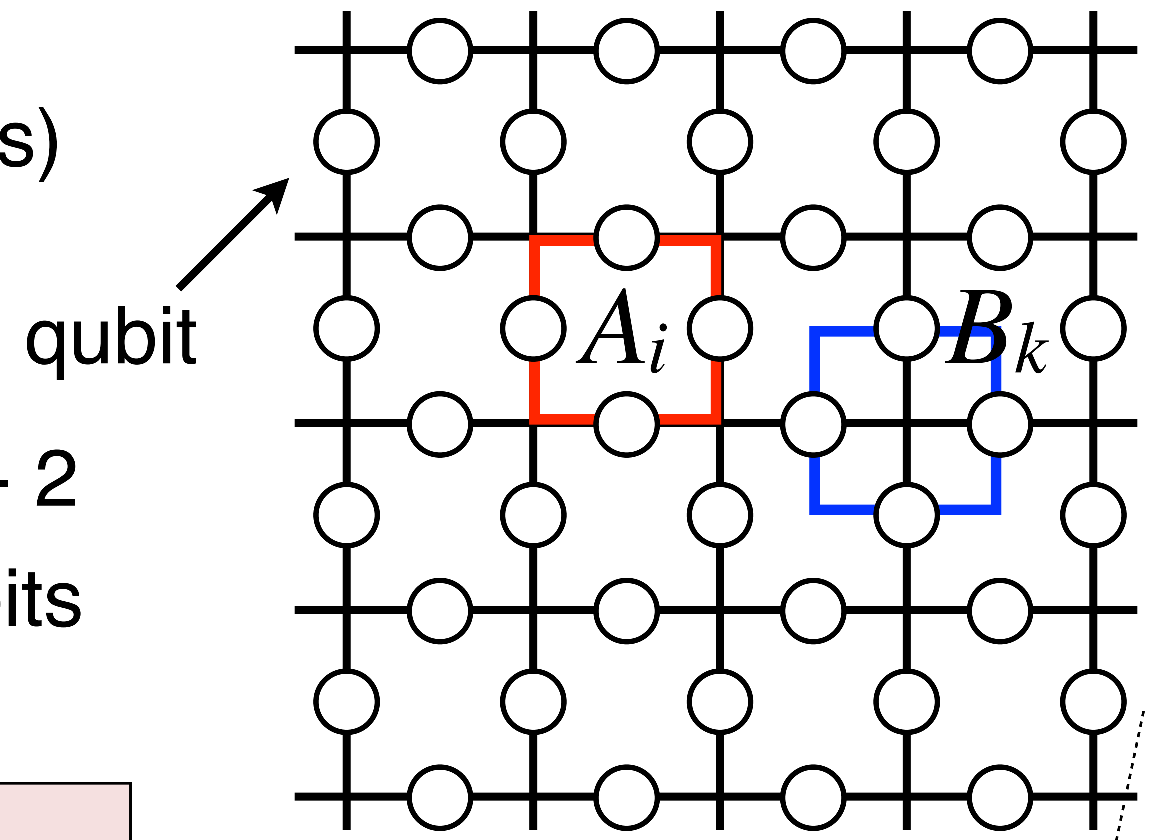
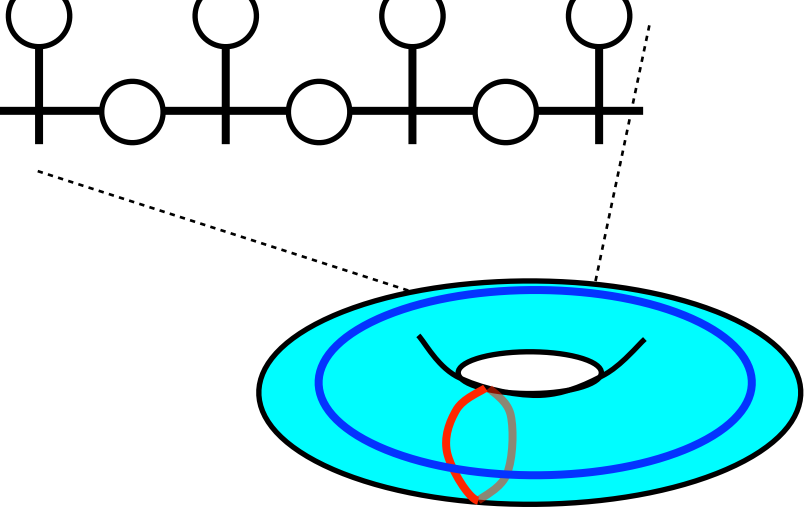


|F| + |V| - |E| = 2 - 2g
Euler characteristic (g = genus)

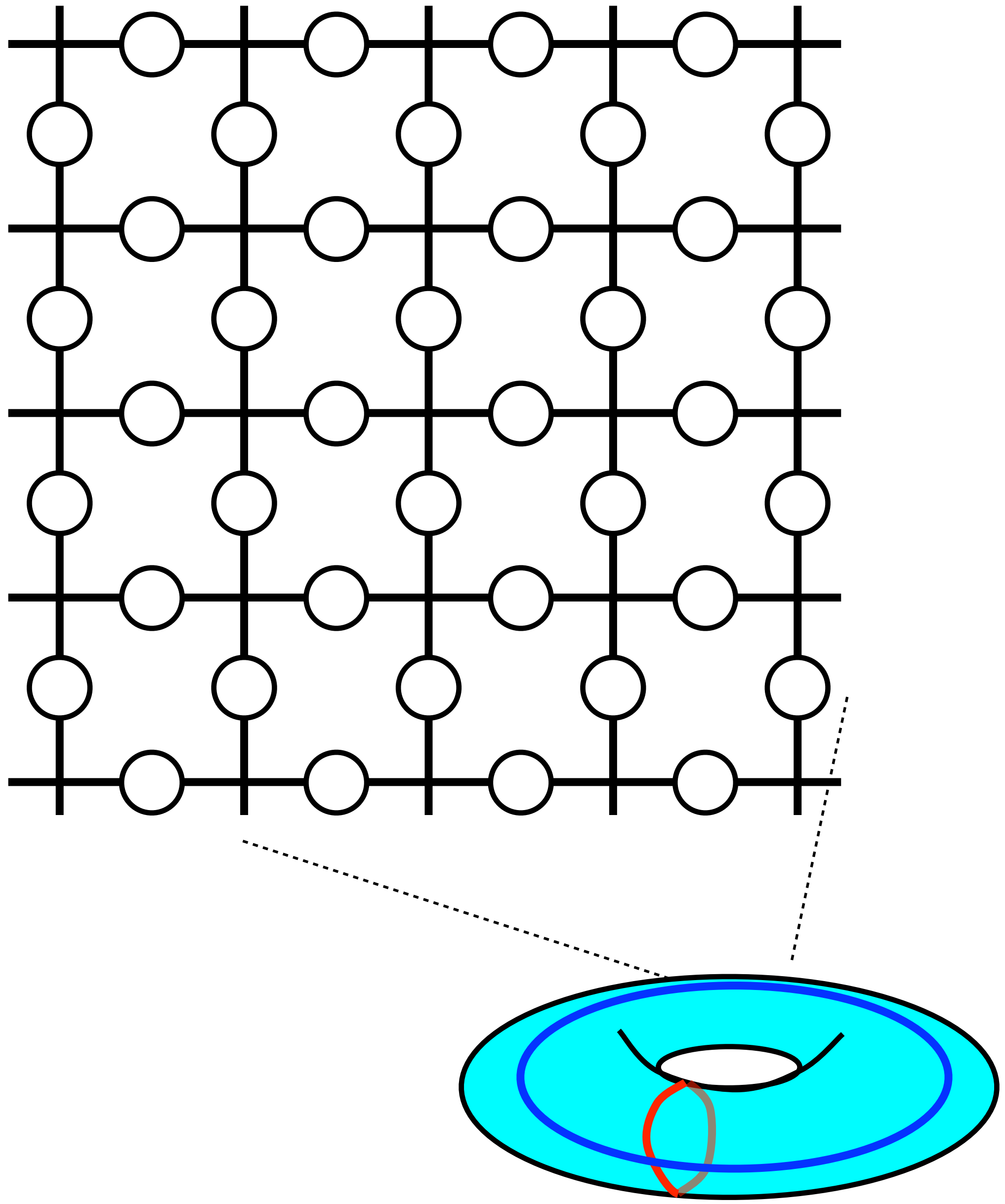
#logical qubits= |E| - (|F| + |V| - 2)=2g

→# of logical qubit = 2g



Logical operators

Logical operators:
commute with and independent of stabilizer operators



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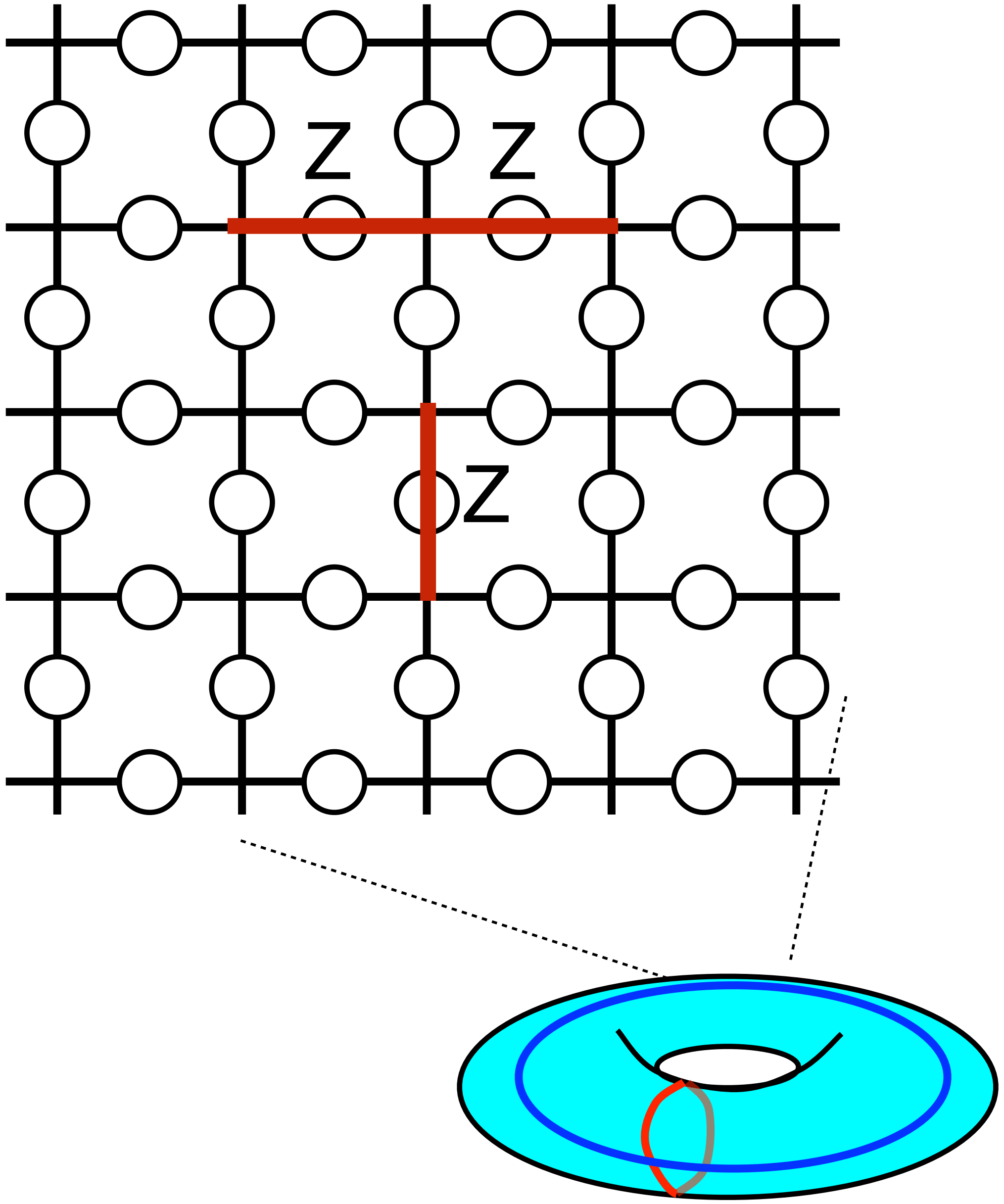

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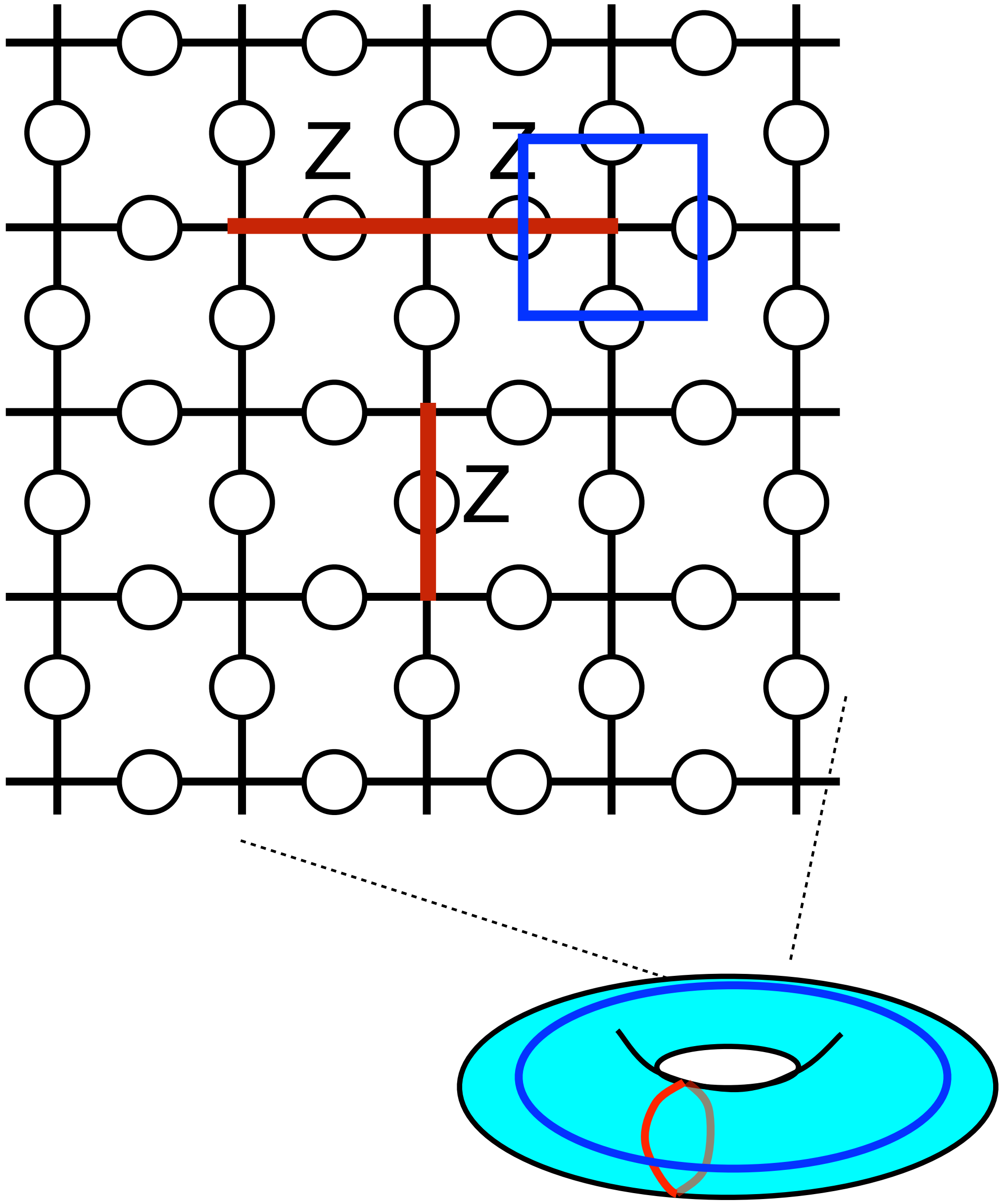

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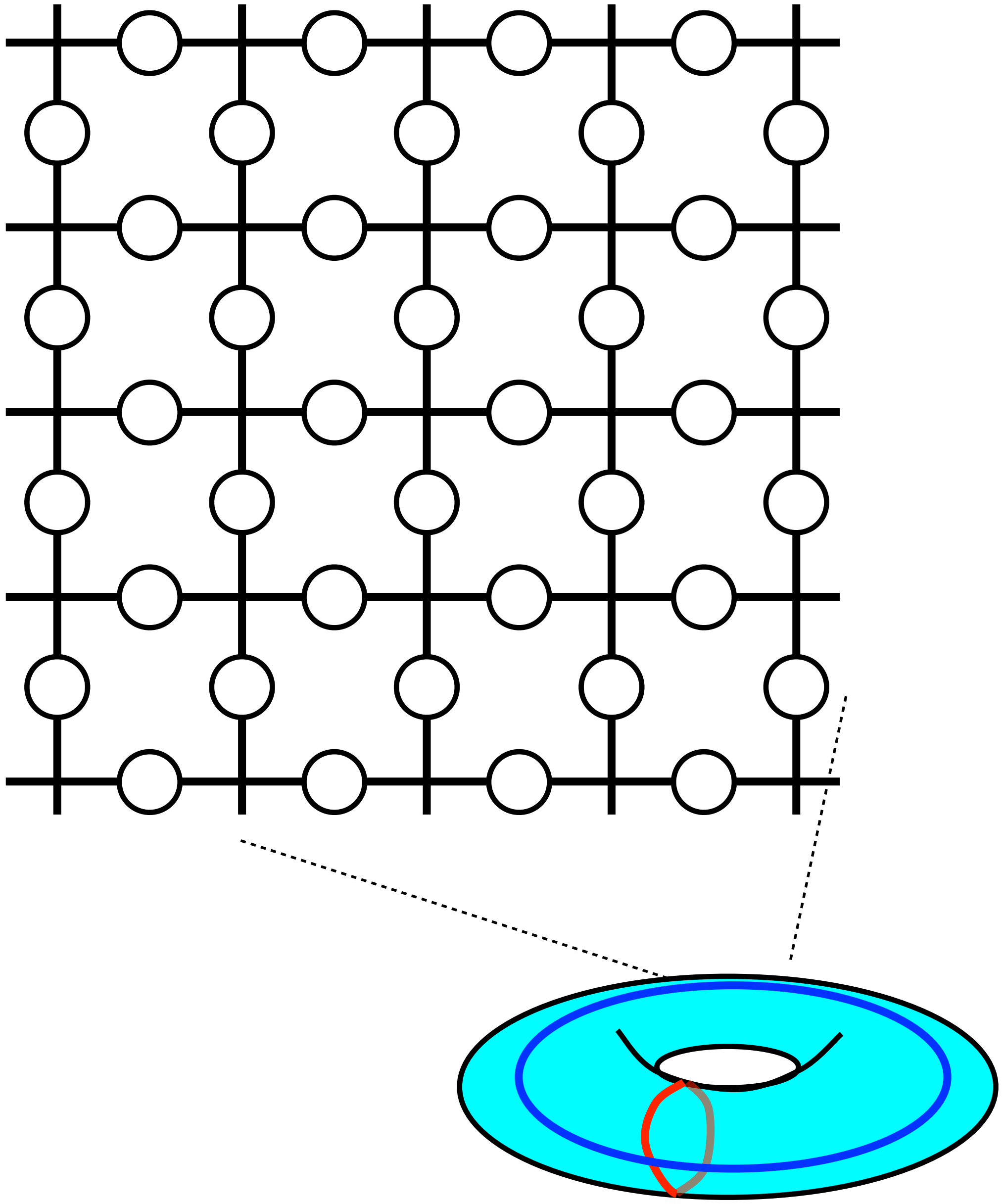

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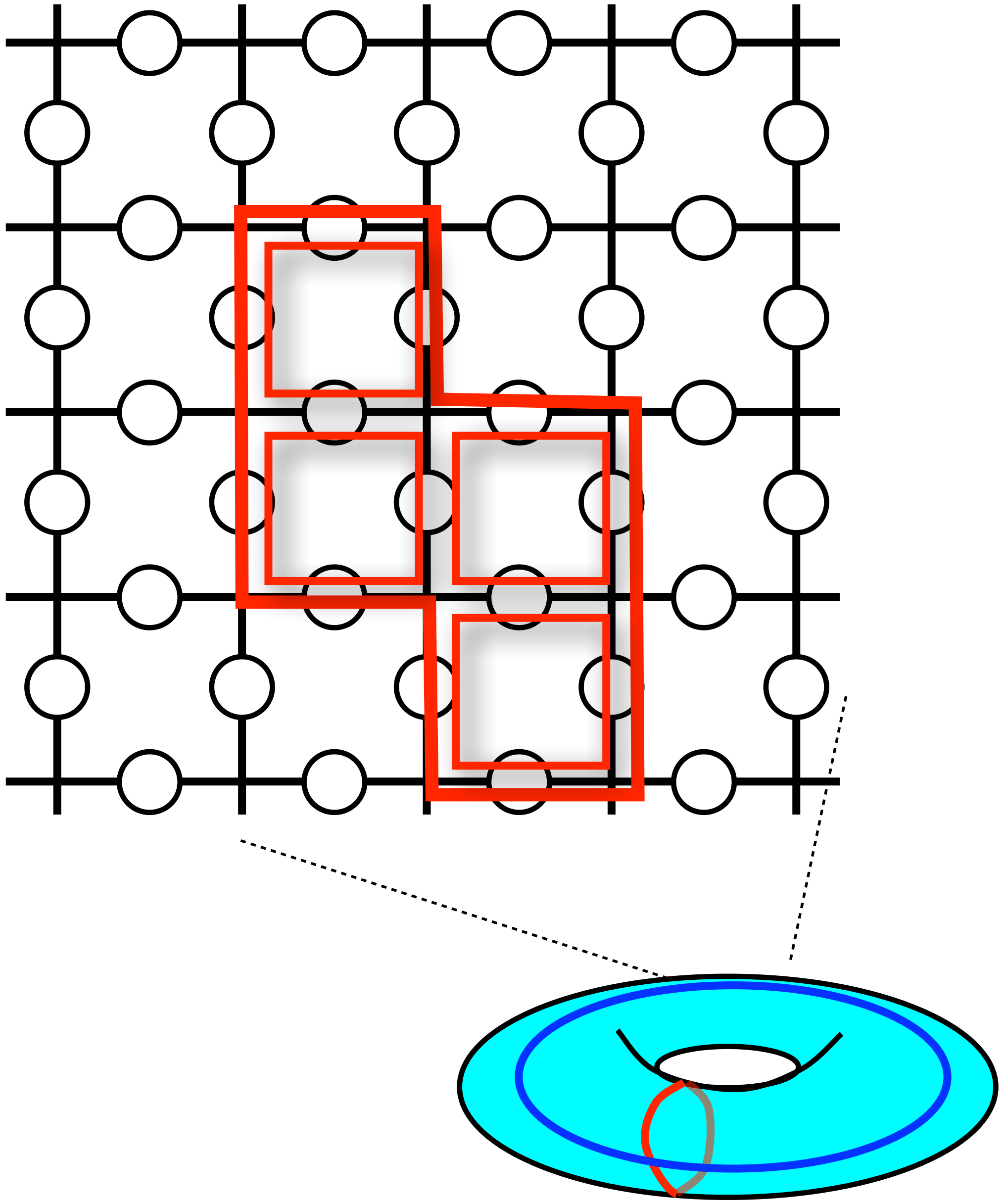
- chains with the ends → error
- trivial cycle operator → stabilizer operator



Logical operators

Logical operators:
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- chains with the ends → error
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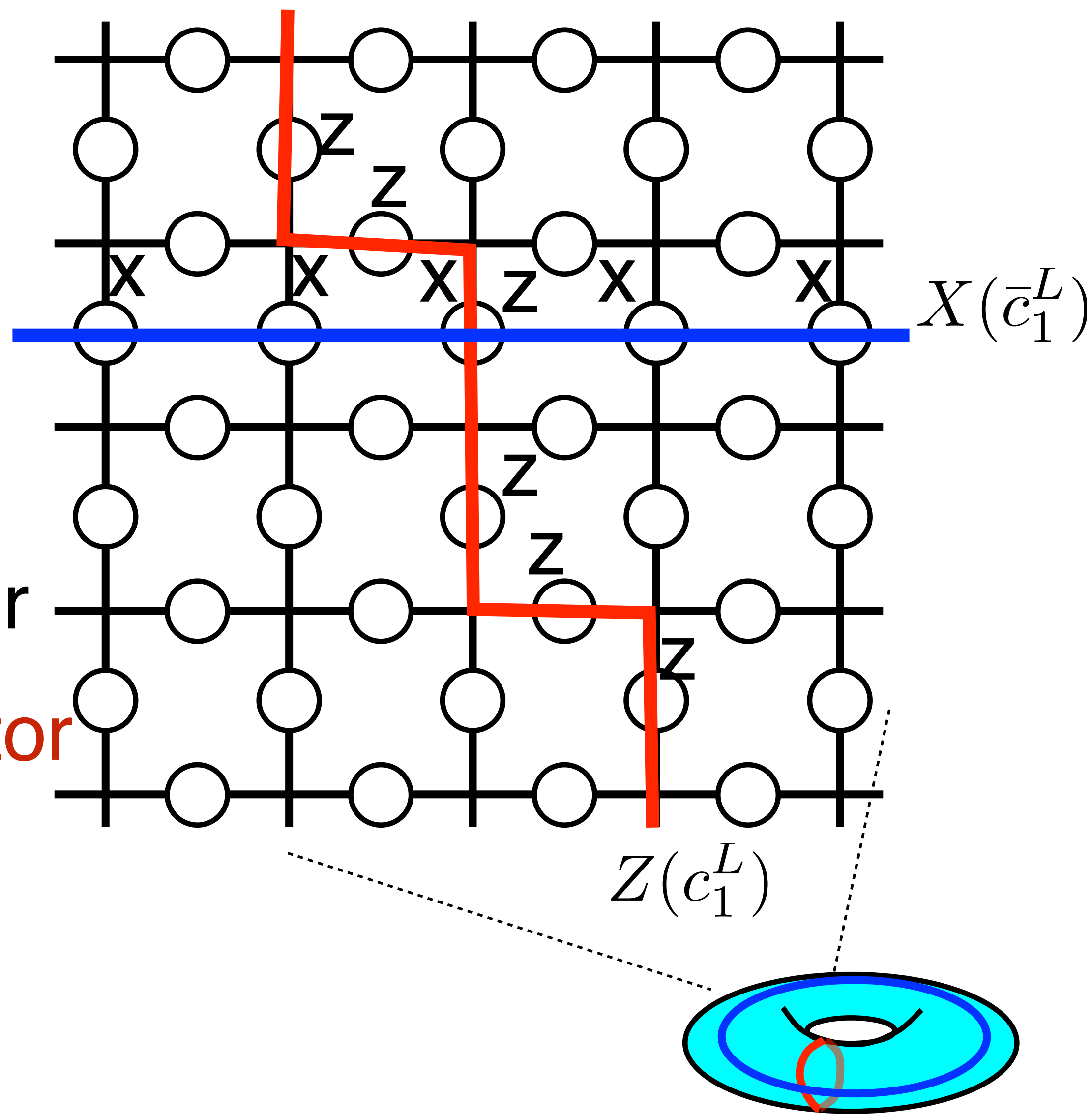
- nontrivial cycle operator → logical operator



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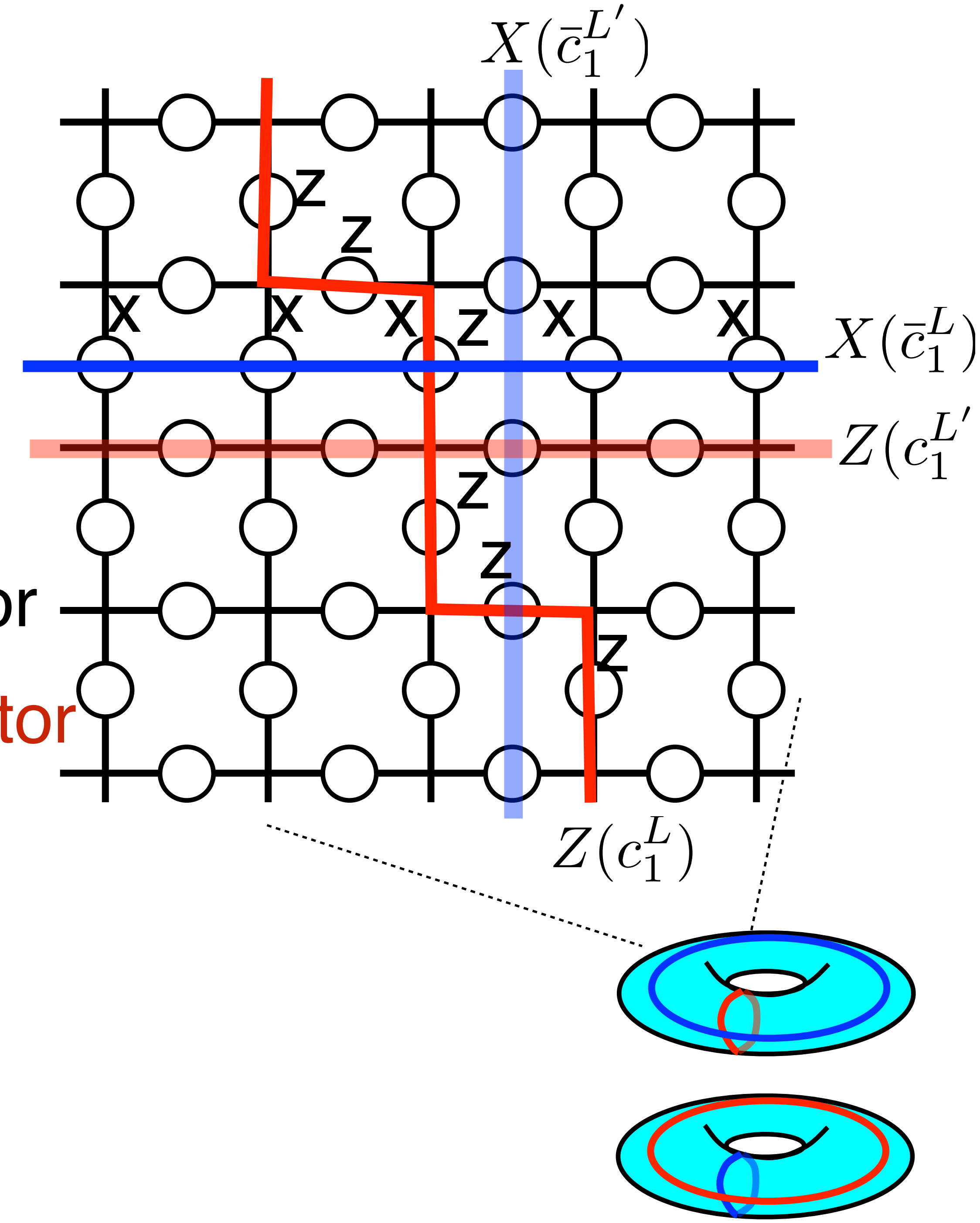

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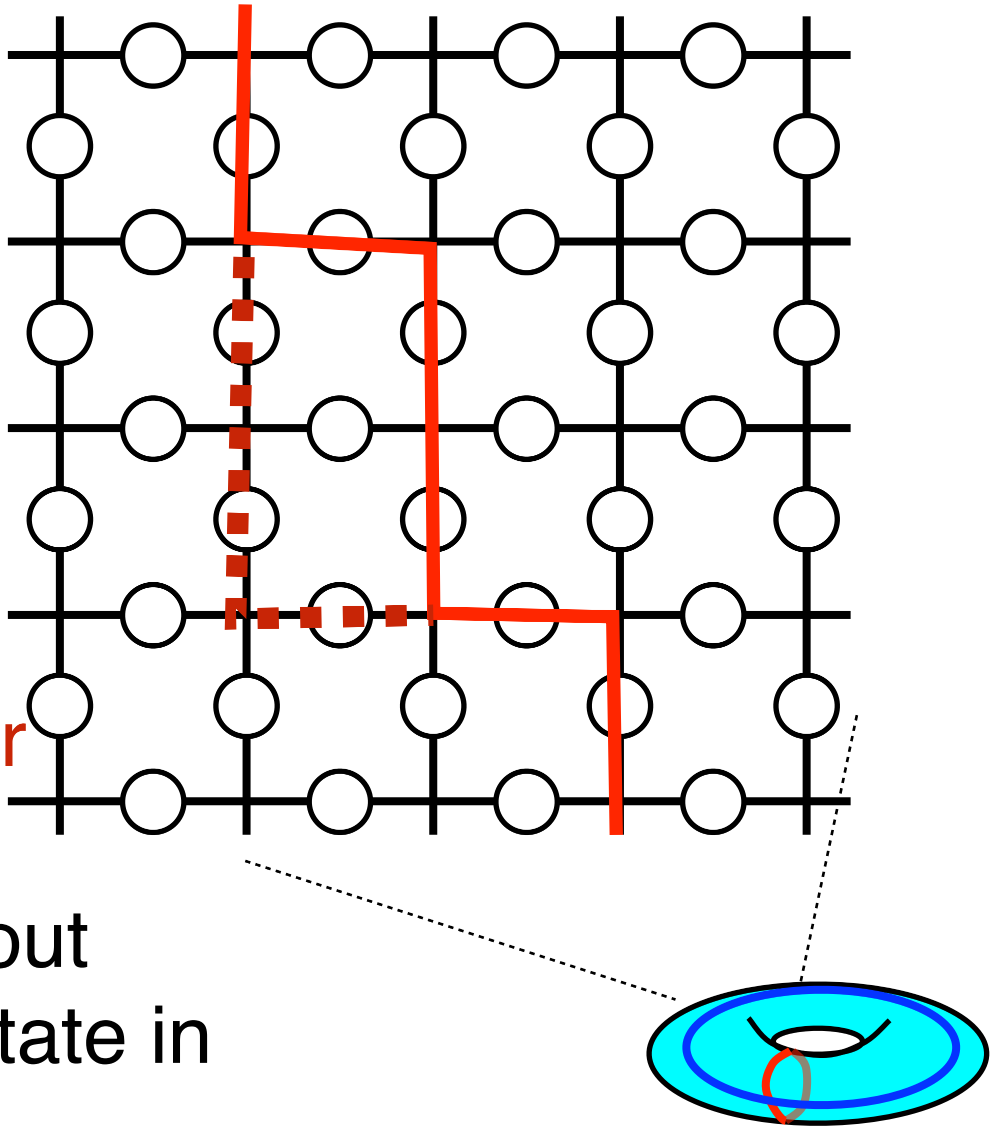


Even if a logical operator is deformed without changing its topology, it acts on the code state in the same way.

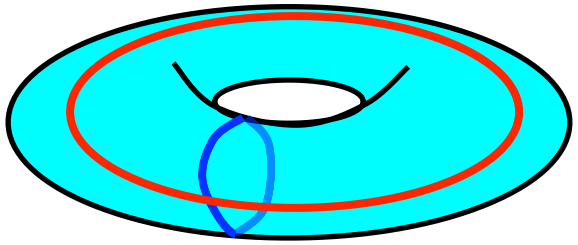
→ The action of logical operators is characterized by topology (homology class).

Toric code Hamiltonian[Kitaev97]

$$H = -J \left(\sum_i A_i + \sum_k B_k \right)$$

stabilizer generators

- The g.s. is 4-fold degenerated.
- Robust against “any” local perturbation. In order to act on the g.s. nontrivially, we need a nonlocal operator wrapping around the torus.

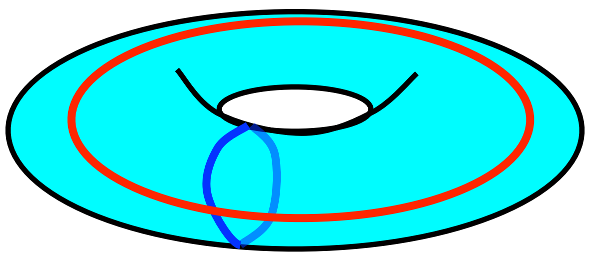


→ **topological order**

Stability against perturbations

local field terms

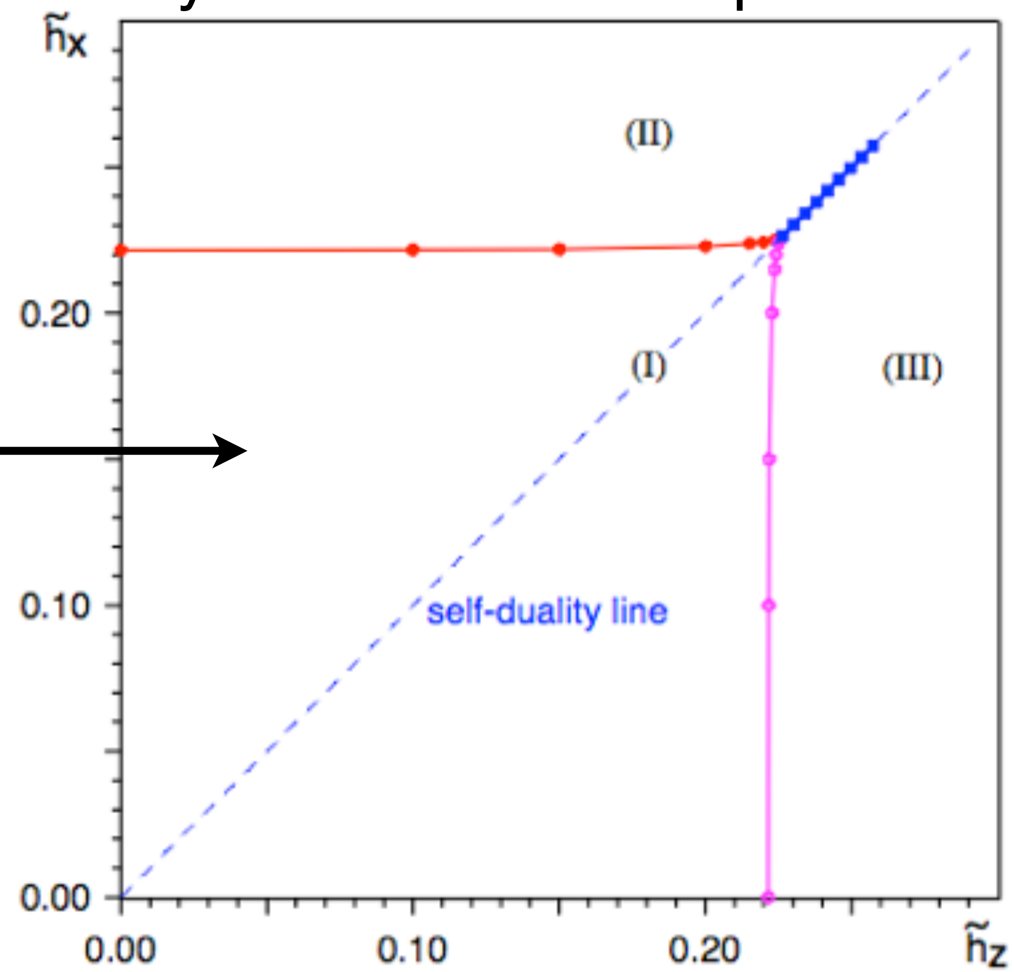
$$H = H_{\text{TC}} + h_x \sum_i X_i + h_z \sum_i Z_i$$



quantum/classical mapping
by Trotter-Suzuki expansion

Z2 Ising gauge model
(dual of 3D Ising model)

topologically ordered
(Higgs phase)



Tupitsyn *et al.*, PRB **82**, 085114 (2010)

Dictionary for QECC and Topological order

quantum error correction codes	topologically ordered system
code subspace	ground state degeneracy

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Toric code (surface code)	Kitaev model

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classical repetition code (can correct either *X* or *Z* errors)	Ising model (non-topological-ordered) →SPT

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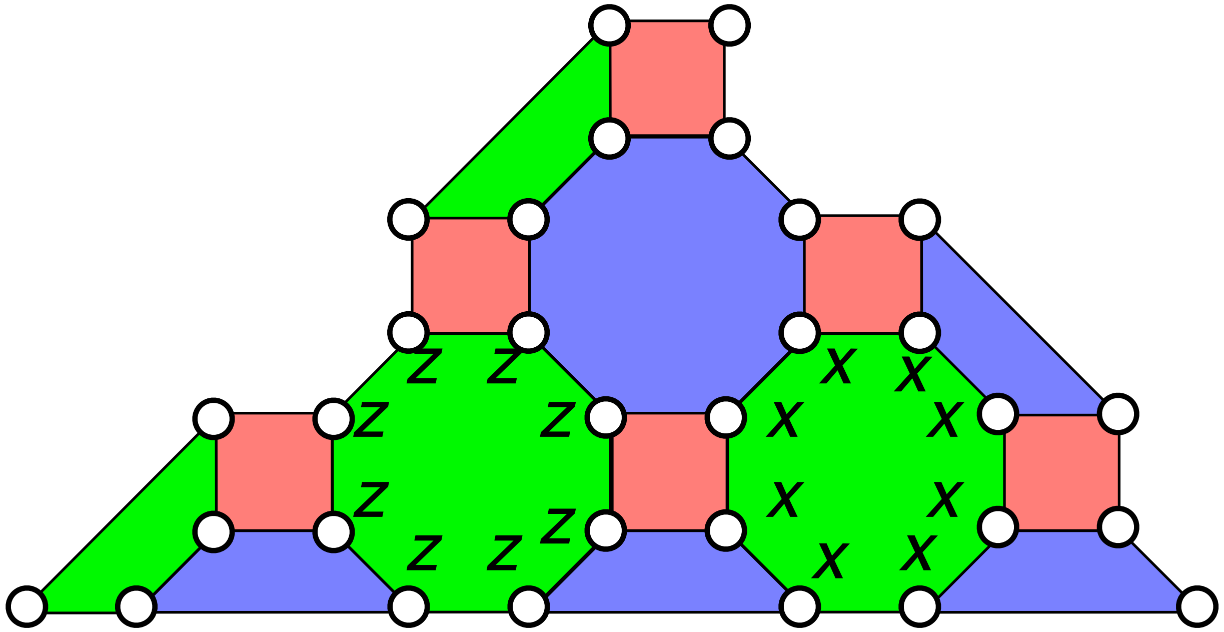
→ thermal stability/ information capacity of discrete systems/ exotic topologically ordered state (fractal quantum liquid)

Topological color codes



Topological color code:

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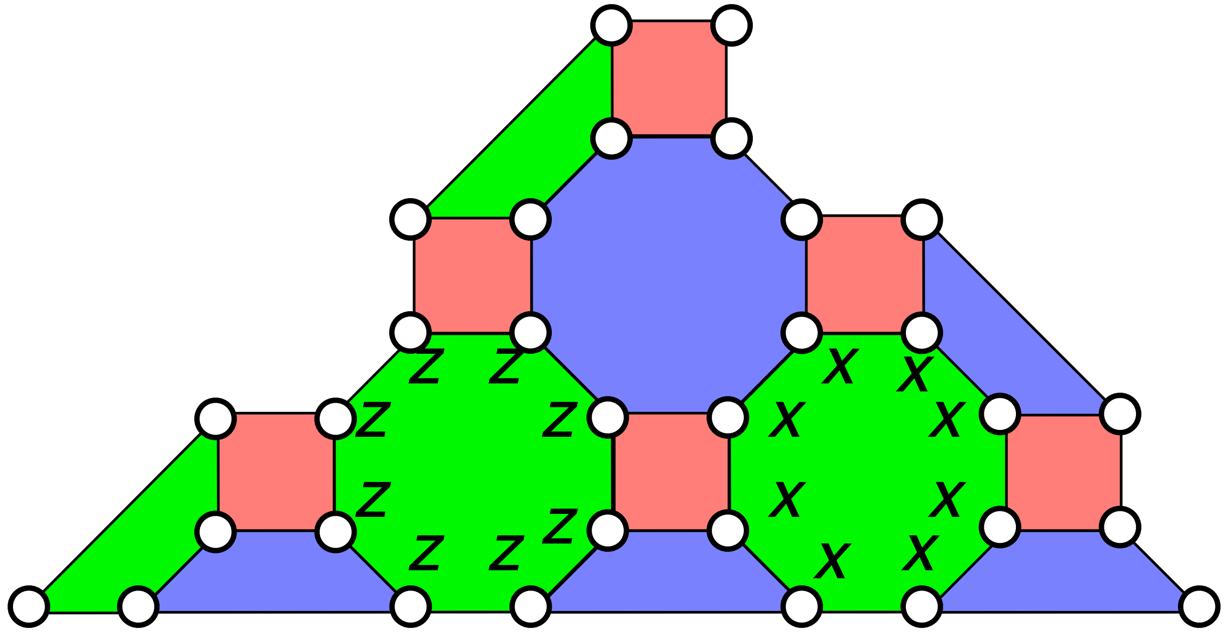
Topological color codes



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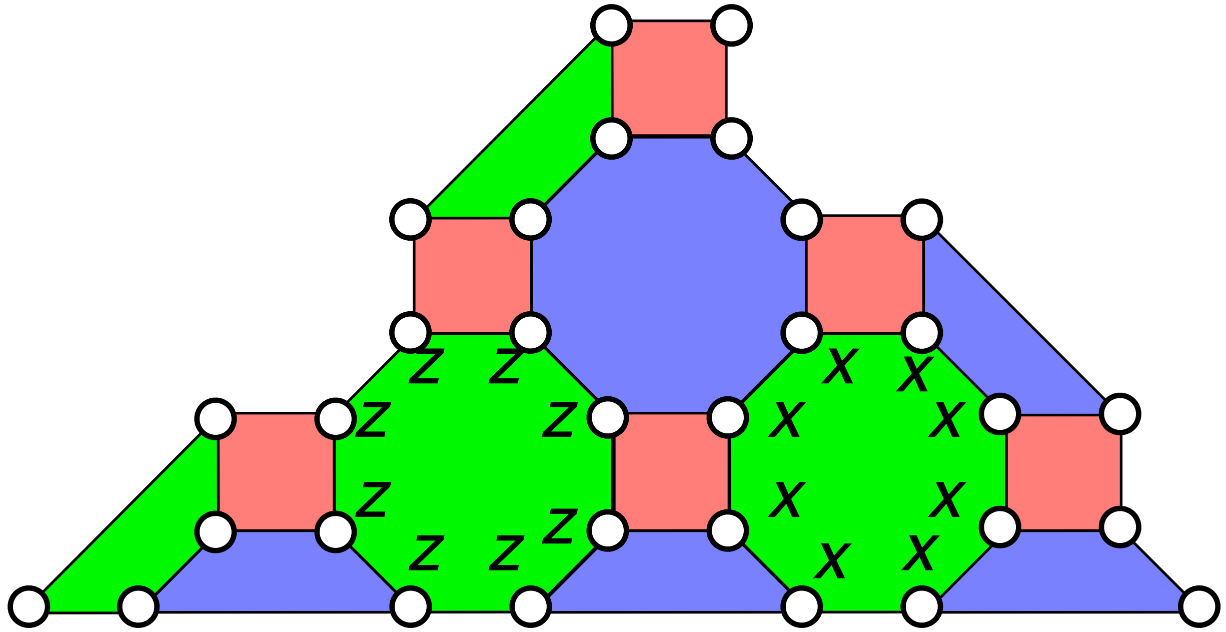

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Any local and translationally invariant topological stabilizer (Hamiltonian) codes in 2D can be classified into multi-copies of toric codes with local unitary operations.

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$$H = -J\left(\sum_i A_i + \sum_k B_k\right)$$

stabilizer generators

- The g.s. is 4-fold degenerated.
- Robust “any” local perturbation. In order to act on the g.s. nontrivially, we need nonlocal operators wrapping around the torus.

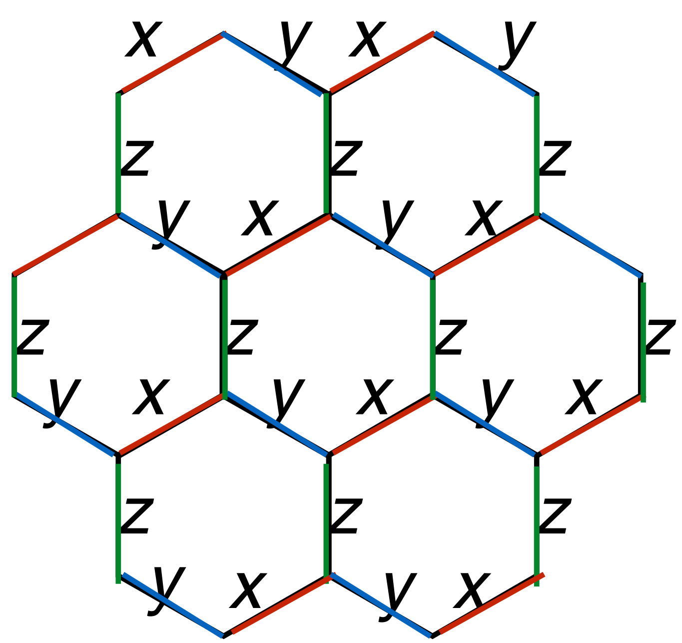
→ **topological order**

Kitaev's honeycomb model

A. Kitaev, Ann. Phys. 321, 2 (2006)

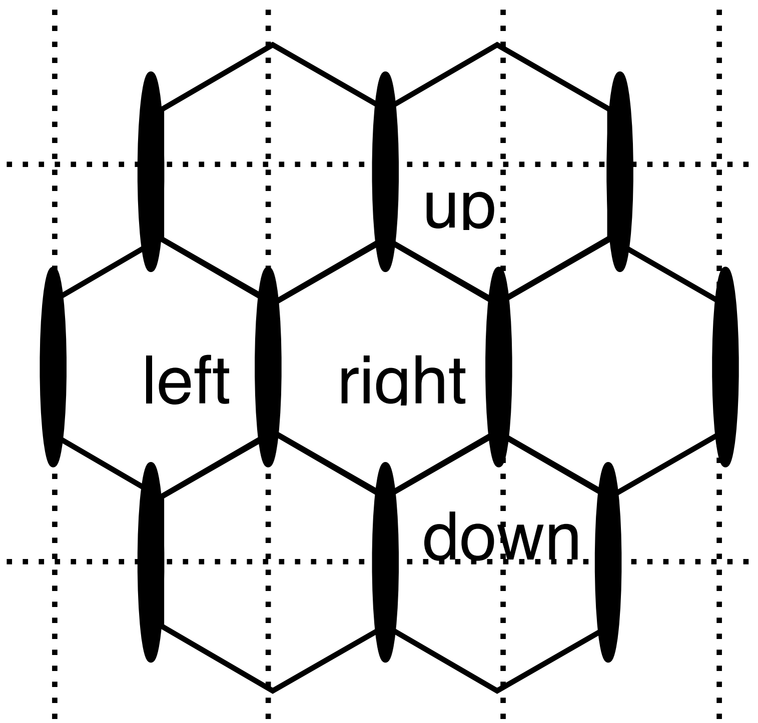
Honeycomb model:

$$H_{\rm hc} = -J_x \sum_{x\text{-link}} X_i X_j - J_y \sum_{y\text{-link}} Y_i Y_j - J_z \sum_{z\text{-link}} Z_i Z_j$$



dimerization

$J_x, J_y \ll J_z$



$$H_{\rm eff} = -\frac{J_x^2 J_y^2}{16|J_z|^3} \sum_p Y_{\text{left}(p)} Y_{\text{right}(p)} X_{\text{up}(p)} X_{\text{down}(p)}$$

local unitary transformation

Toric code Hamiltonian:

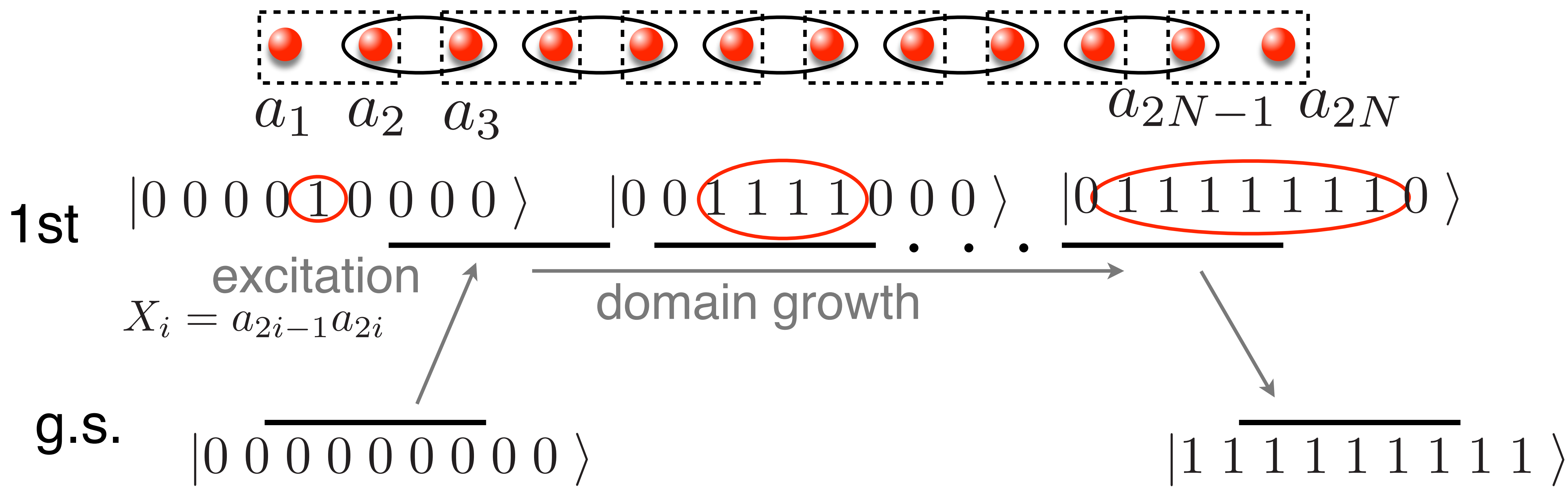
$$H_{\rm TC} = -J \sum_f Z_{l(f)} Z_{r(f)} Z_{d(f)} Z_{u(f)} - J \sum_v X_{l(v)} X_{r(v)} X_{d(v)} X_{u(v)}$$

A. Kitaev, Ann. Phys. 303, 2 (2003)

Is topological order robust even at finite temperature? or under decoherence?

Thermal instability of Majorana chain

Unpaired Majorana fermion:



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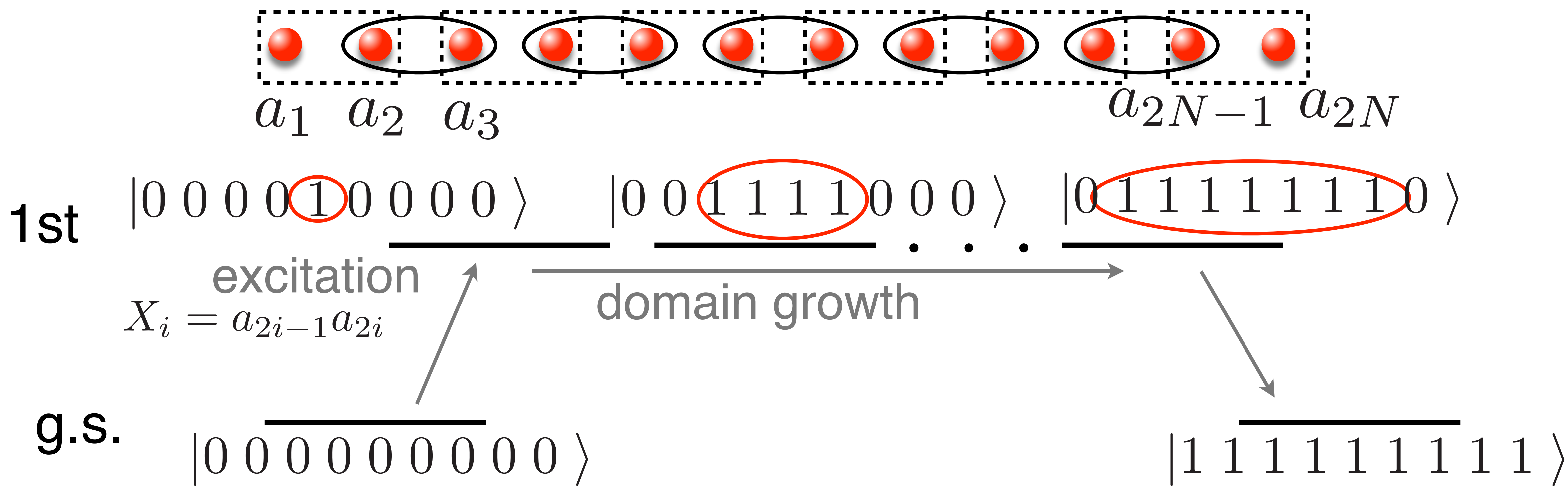
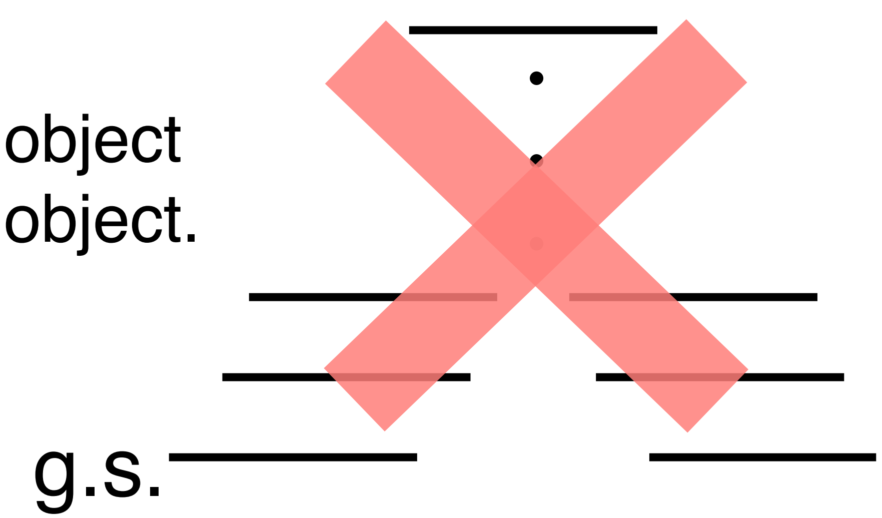


Excitation (domain-wall) is a point-like object
→ logical operator is a string-like (1D) object.
→ no energy barrier



Thermal instability of topological order in 2D

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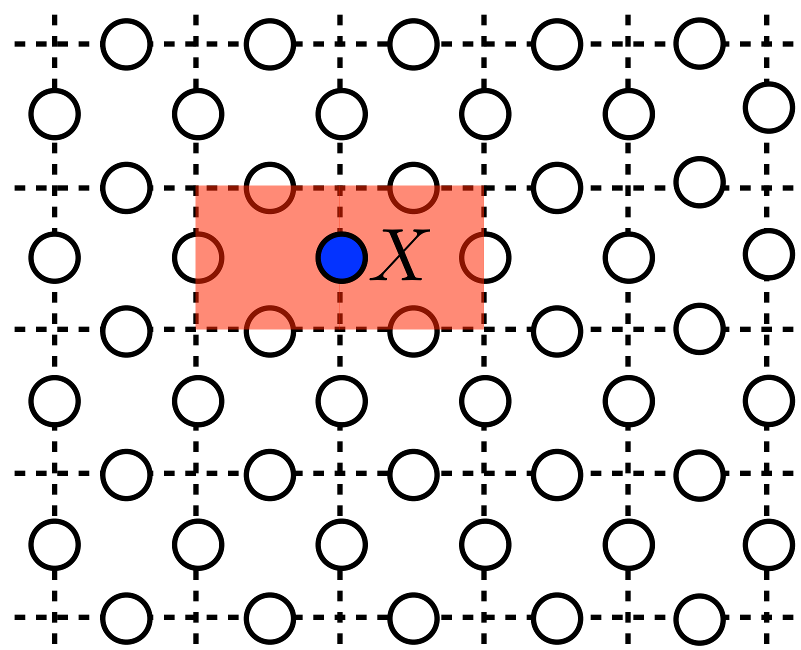
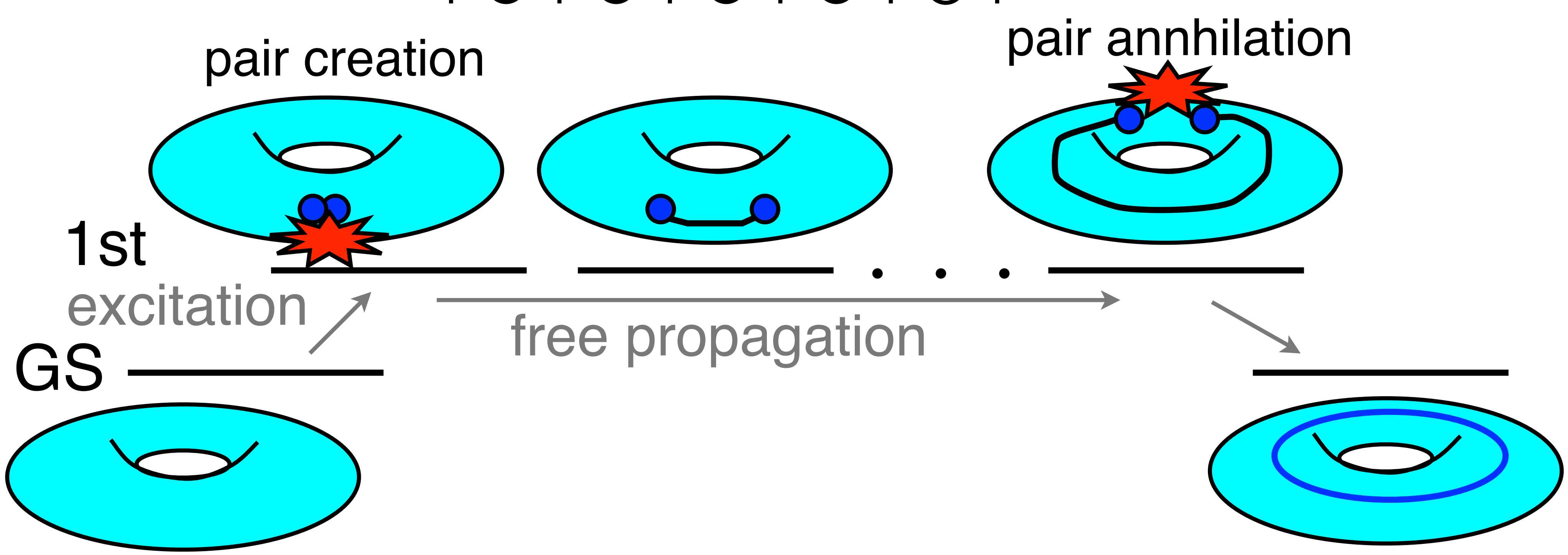


Anyon can move freely without any energetic penalty.



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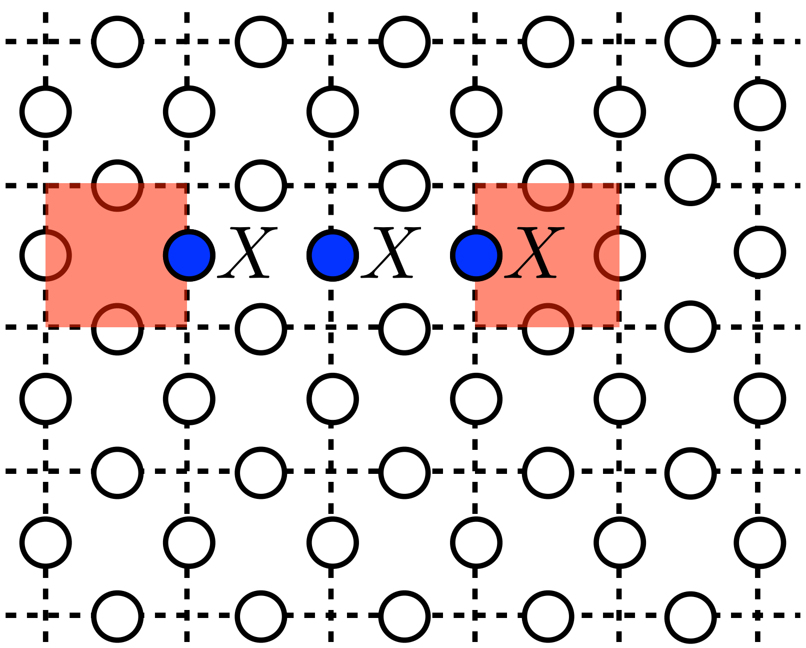

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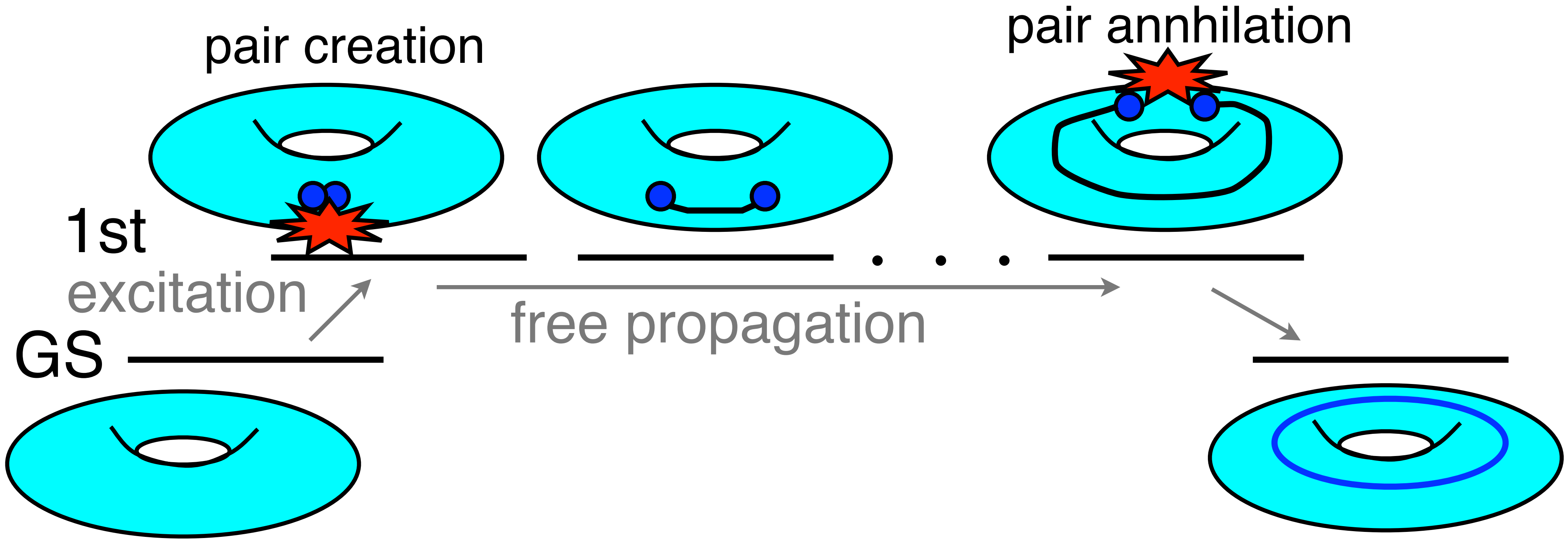

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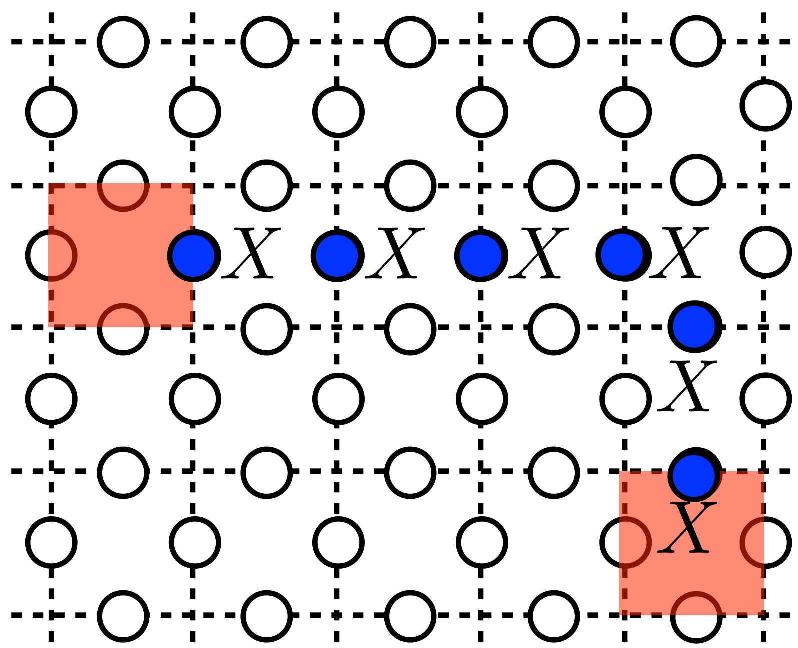

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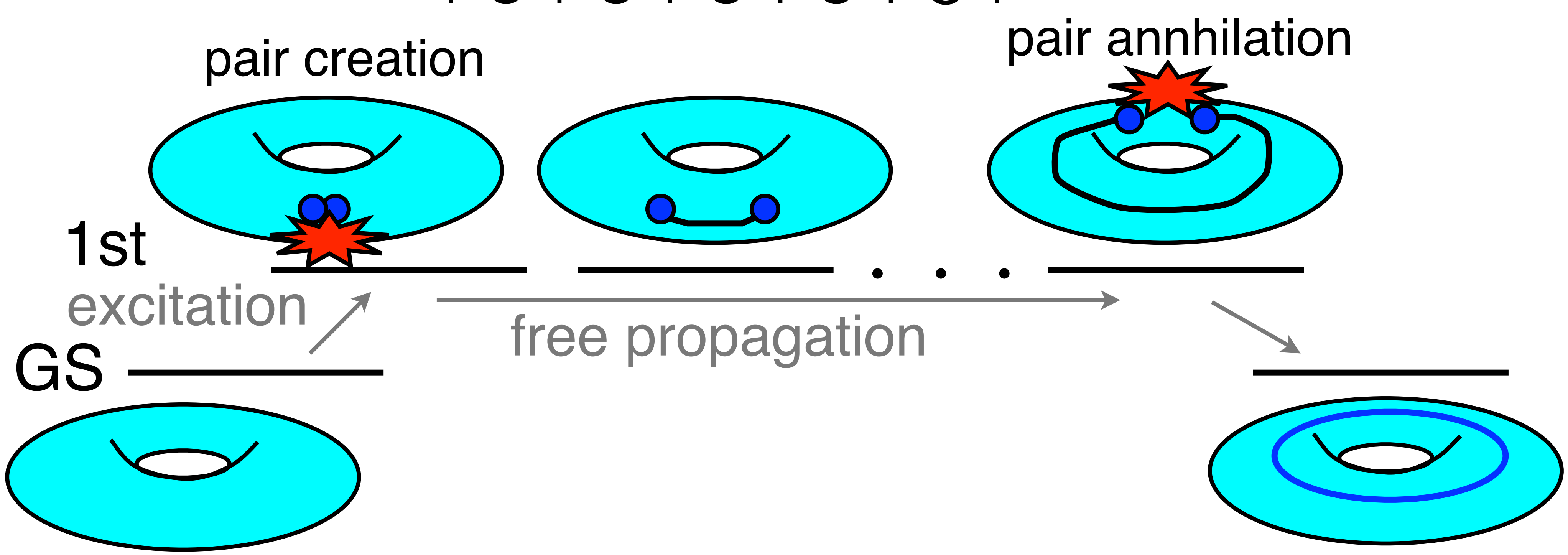

Thermal stability of topological order

More generally...

Topological order in any local and translation invariant stabilizer Hamiltonian systems in 2D and 3D do not have thermal stability.

2D: S. Bravyi and B. Terhal, New J. Phys. **11**, 043029 (2009).
3D: B. Yoshida, Ann. Phys. **326**, 2566 (2011).

← quantum error correction code theory

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Thermally stable topological order (self-correcting quantum memory) in 4D

by E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, J.Math.Phys. **43,** 4452 (2002).

(Excitation has to be two-dimensional object for each non-commuting errors, X and Z. →4D)

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Existence/non-existence of thermally stable topological order (= self-correcting quantum memory) in 3 or lower dimensions is one of the open problems in physics!
(see list of unsolved problem in physics in wiki)

Thermal stability of topological order

More generally...

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← quantum error correction code theory

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(Excitation has to be two-dimensional object for each non-commuting errors, X and Z. →4D)

Existence/non-existence of thermally stable topological order (= self-correcting quantum memory) in 3 or lower dimensions is one of the open problems in physics!
(see list of unsolved problem in physics in wiki)

Non-equilibrium condition (feedback operations) is necessary to observe long-live topological order (many-body quantum coherence) at finite temperature.

Thermal stability of topological order

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This is the reason why we need quantum error correction for quantum computer.

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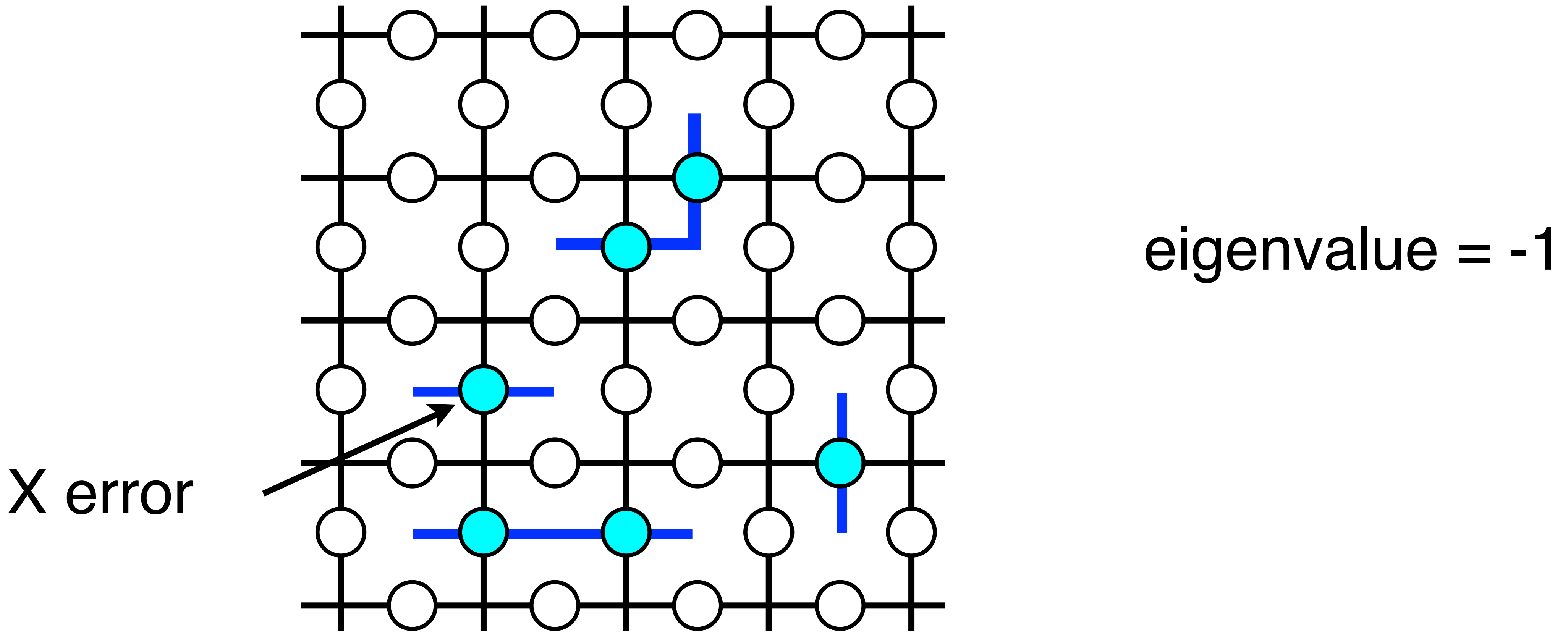
Topological quantum error correction

How errors are detected



Errors are detected at the boundary of the error chain.

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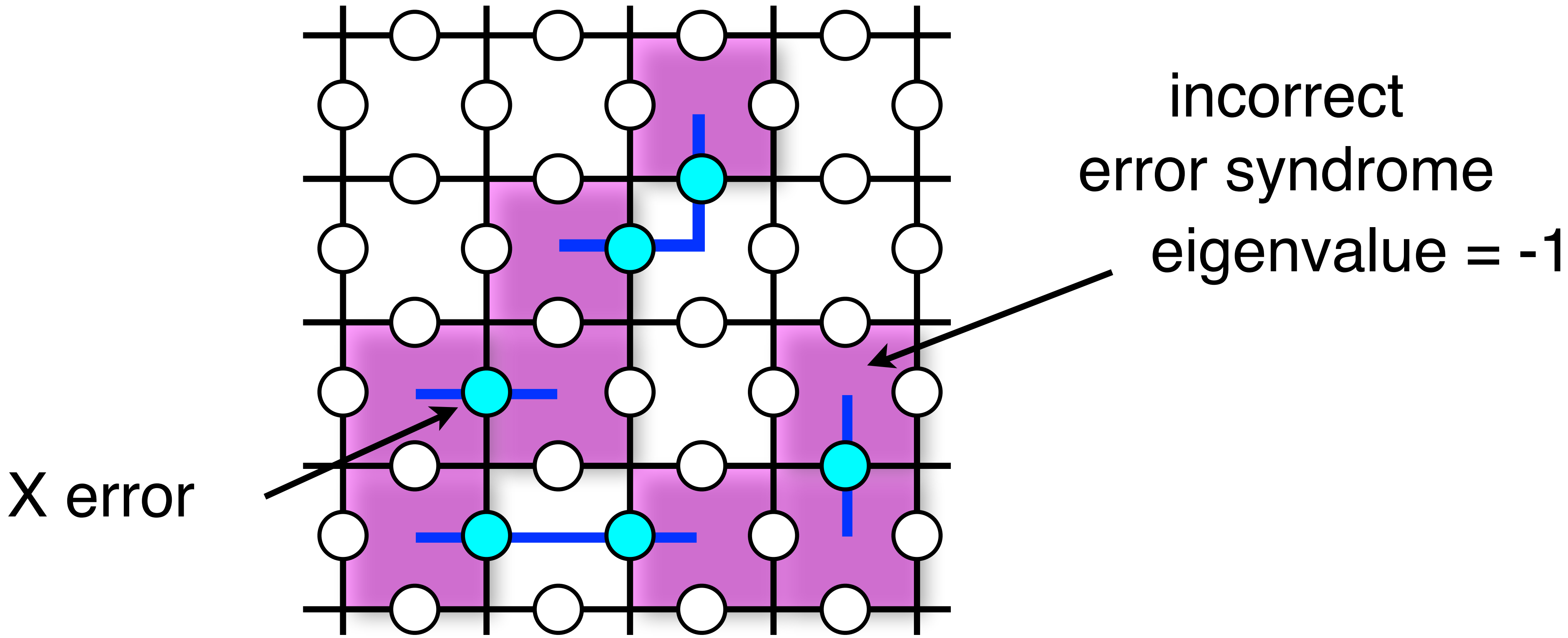
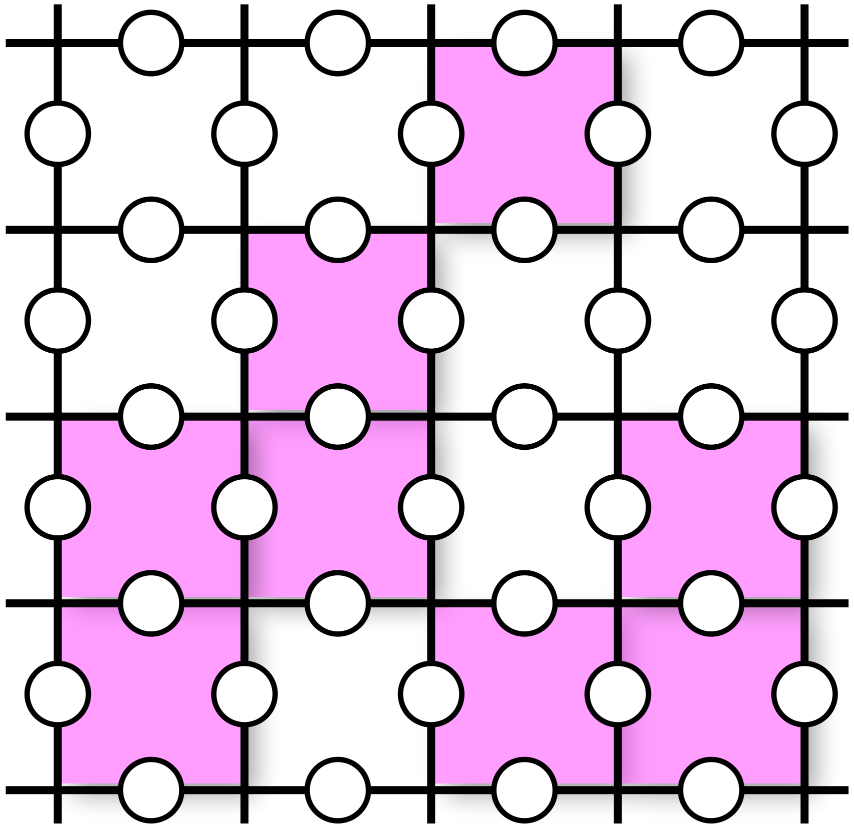


Errors are detected at the boundary of the error chain.

Error estimation

We don’t know about the actual location of the errors.



From the boundary information, we have to estimate the most likely location of the errors.

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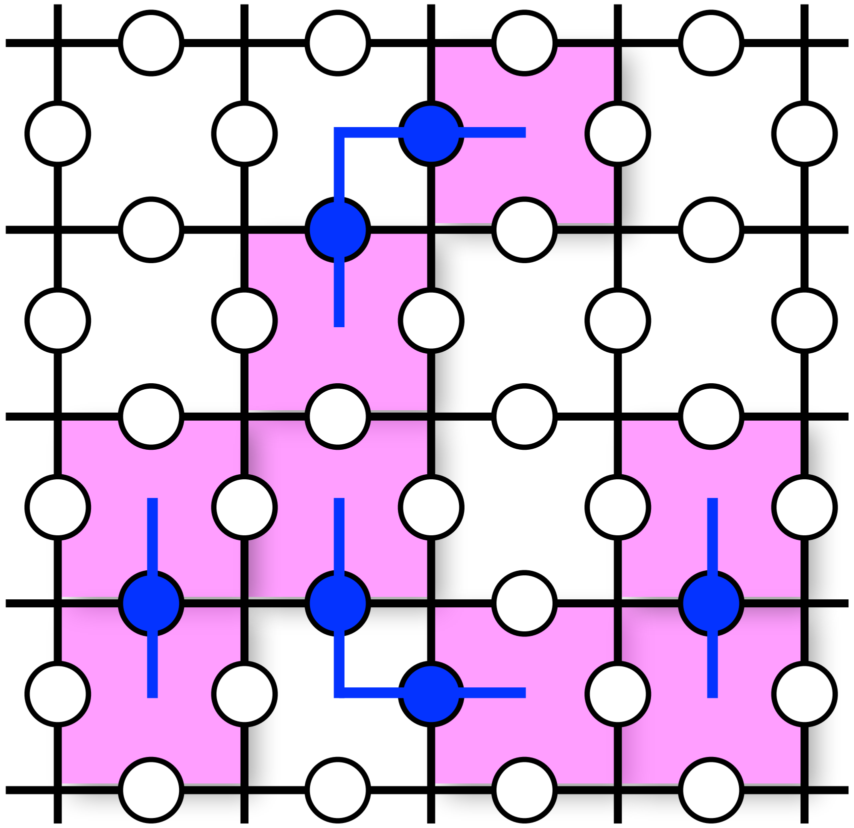

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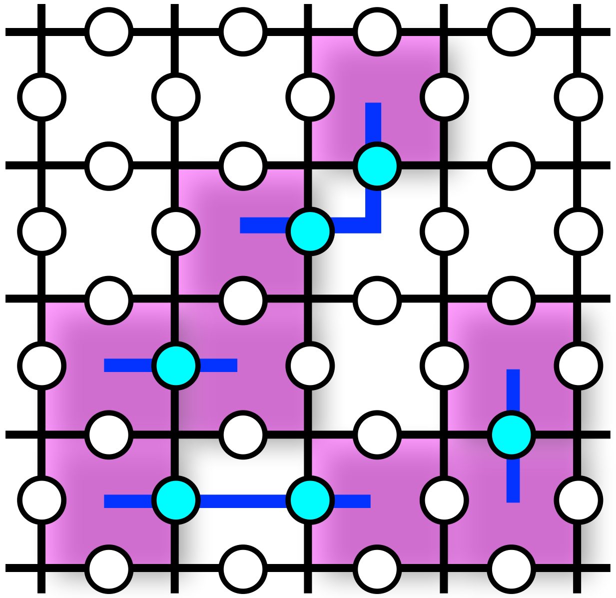
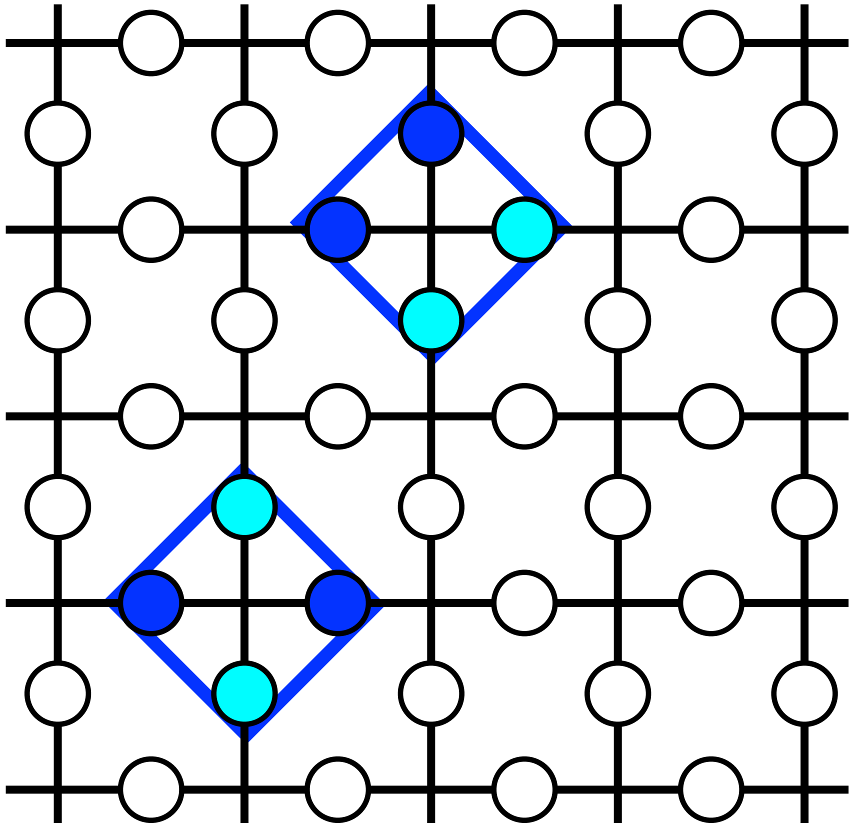
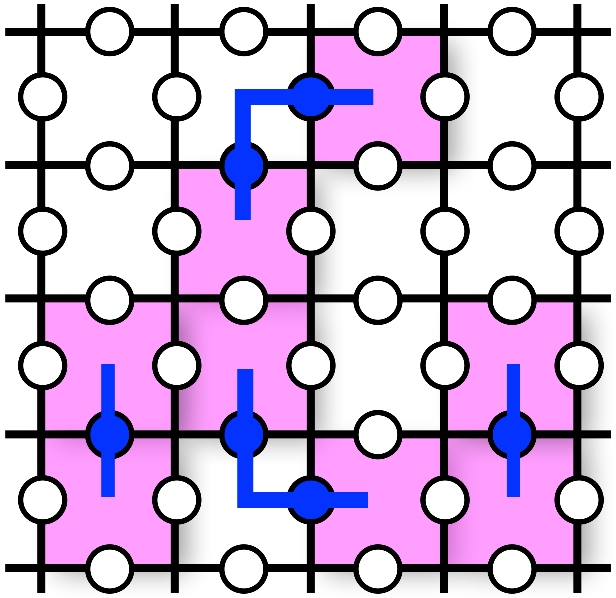
Error estimation

actual errors + estimated errors → trivial cycle



actual errors

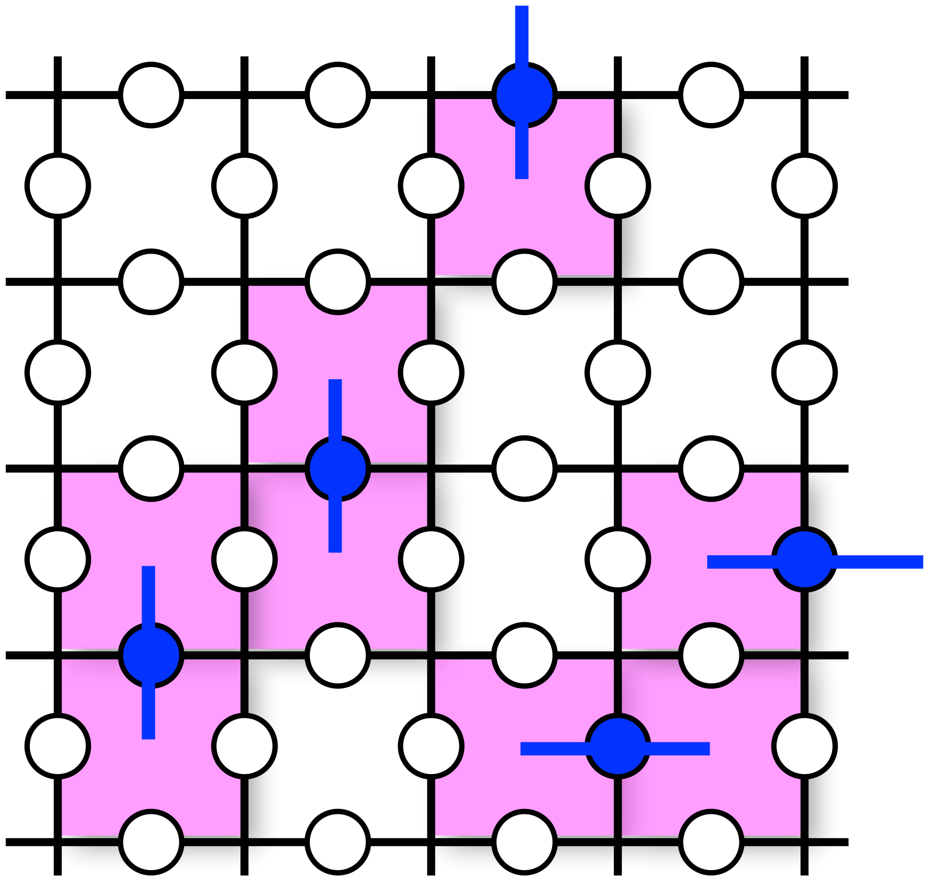




estimated errors

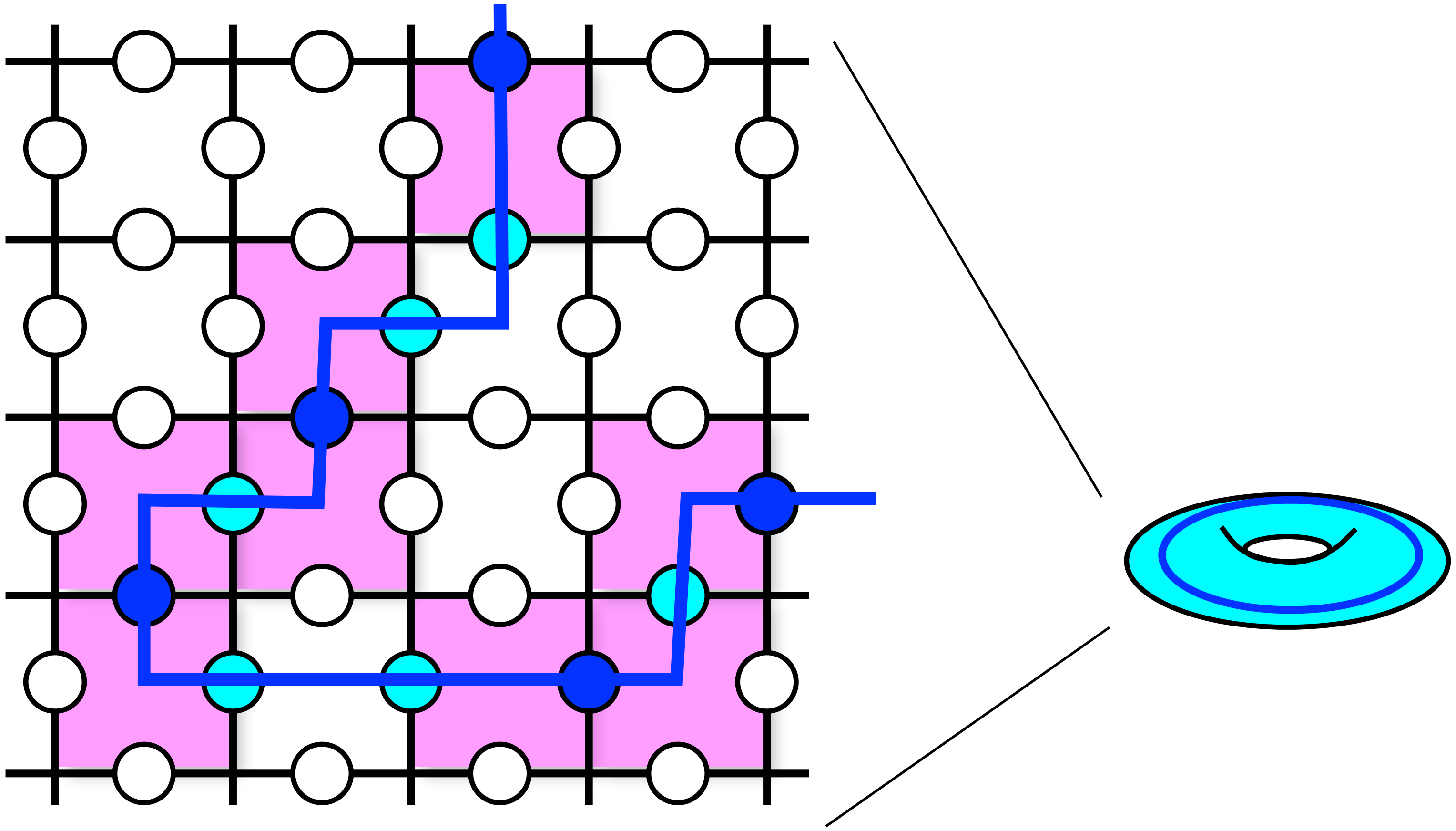
Error estimation

Another estimation of the location of the errors.



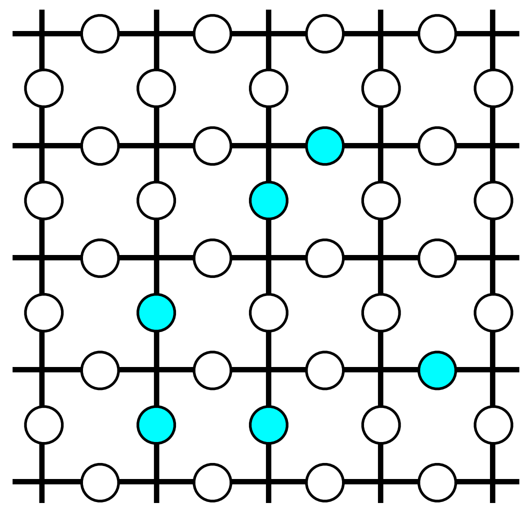
Error estimation

actual errors + estimated errors → non-trivial cycle

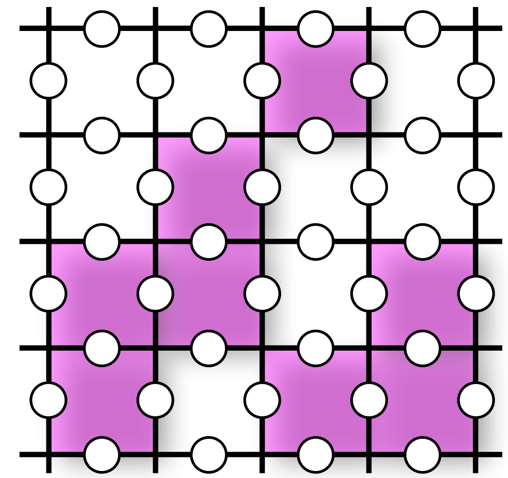


This changes the logical state unintentionally, and hence the error correction fails.

Error estimation



errors

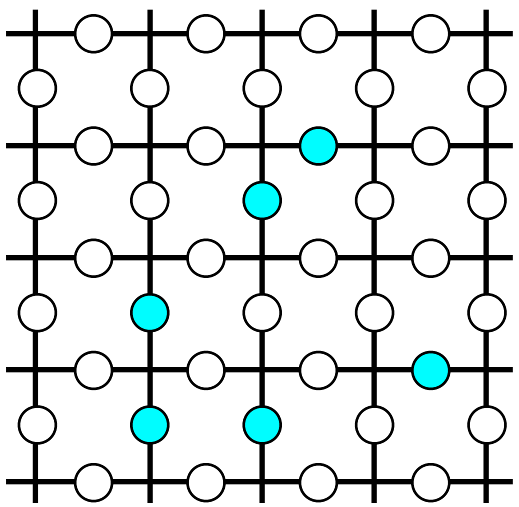


syndrome = boundary of the error chain

→ Maximize the posterior probability conditioned on the given syndrome:

$$\arg\max_{E} p(E|S = \partial E)$$

Error estimation



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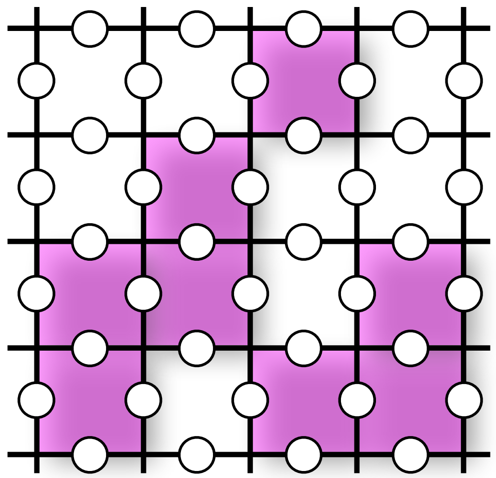

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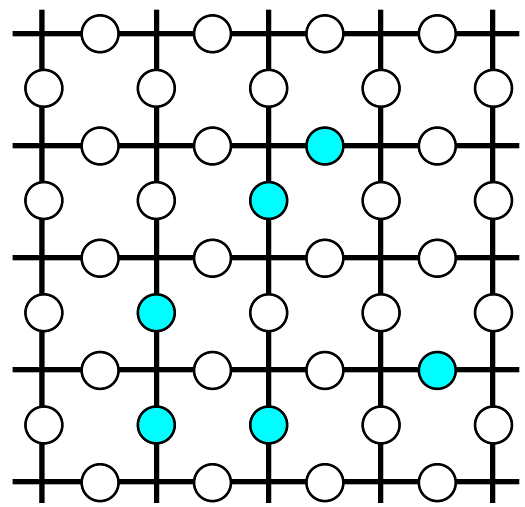
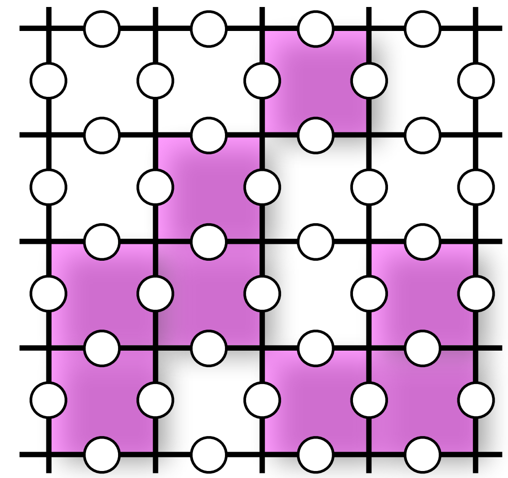


errors



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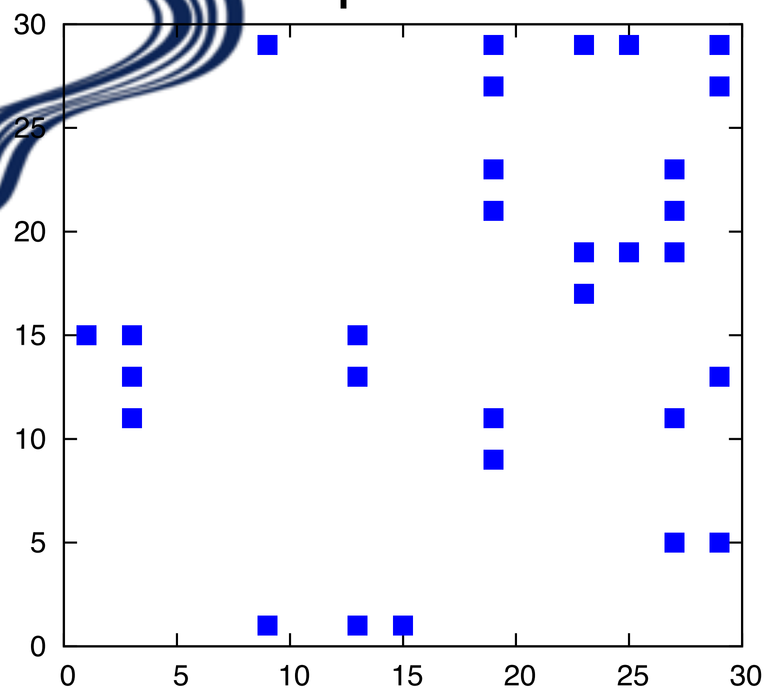
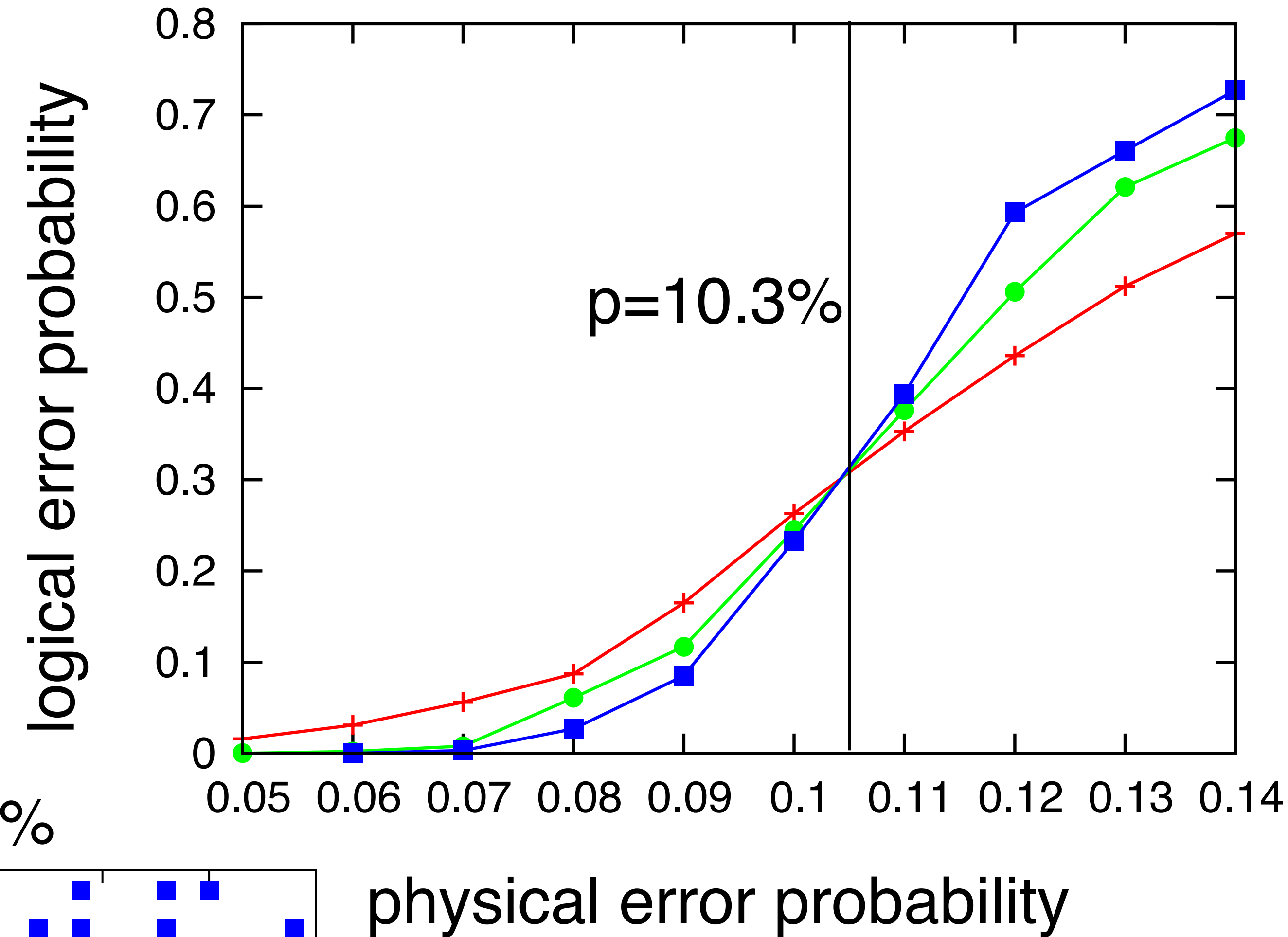
→ minimum-weight perfect match algorithm (**MWPMA**)

E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, J.Math. Phys. **43**, 4452 (2002).

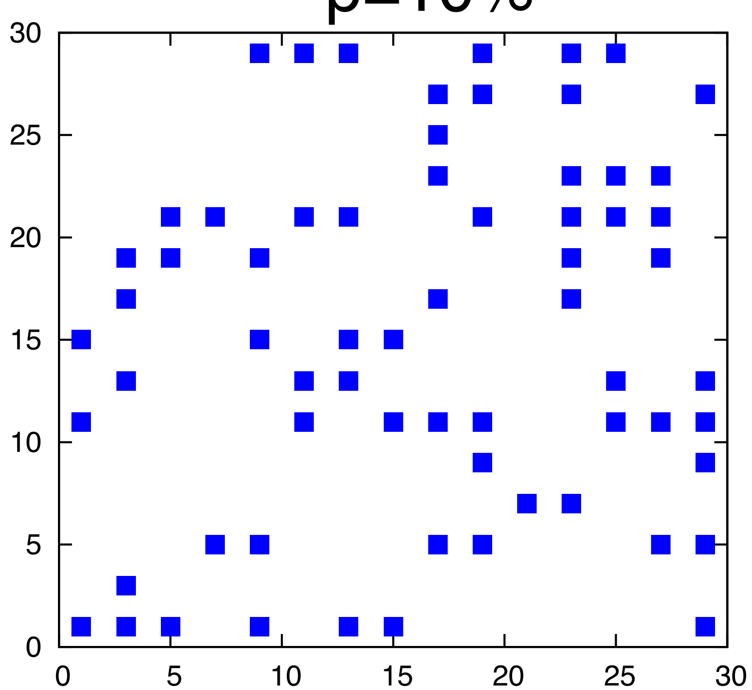
Topological error correction

p=3%

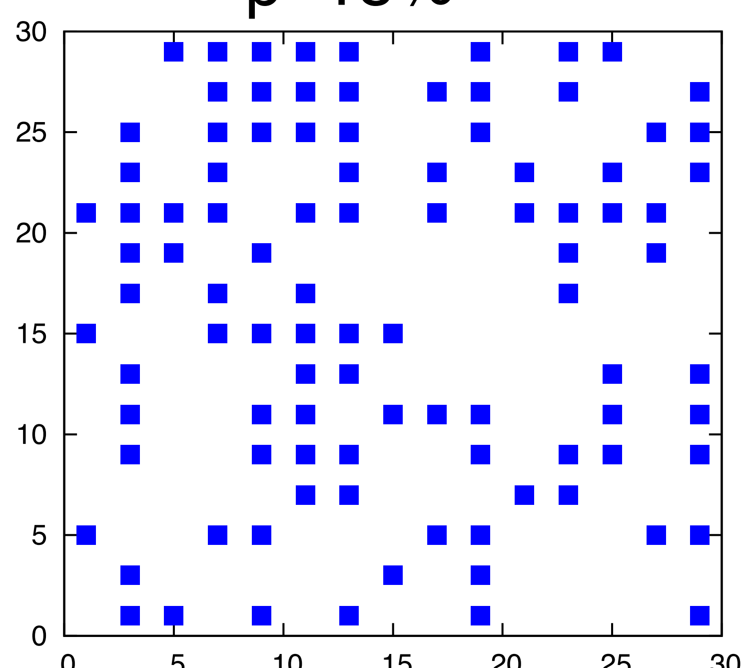




p=10%

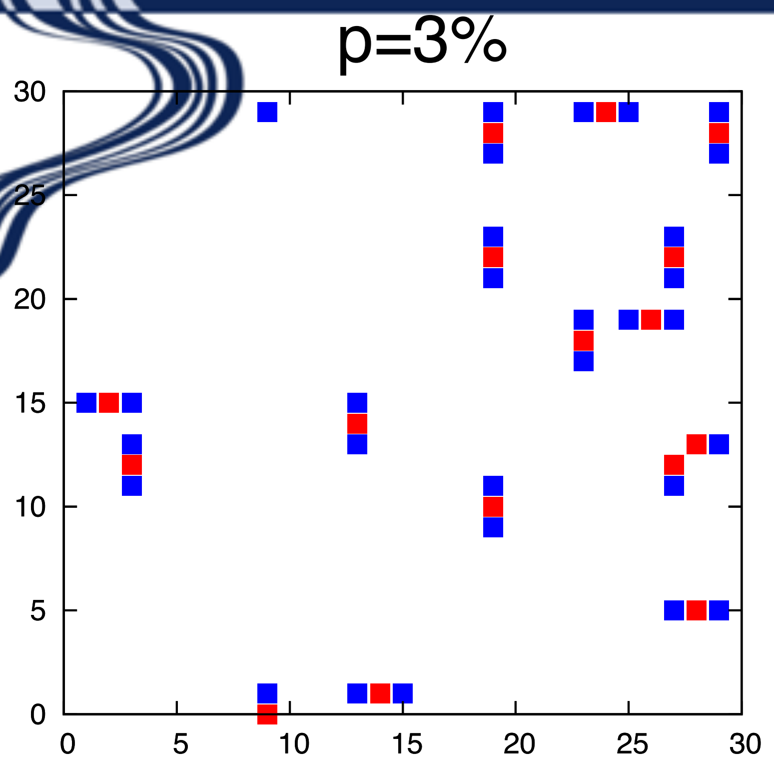


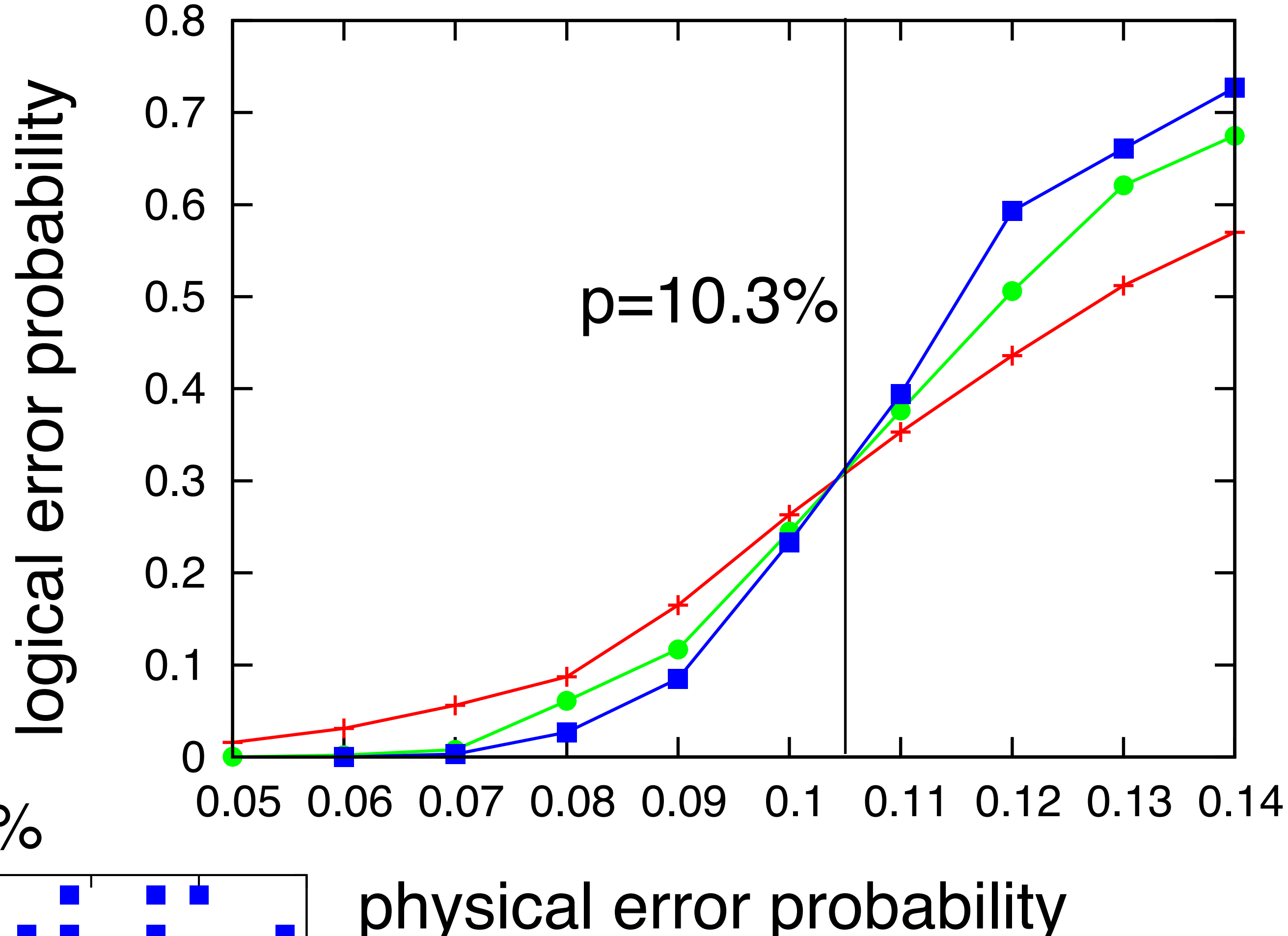
p=15%

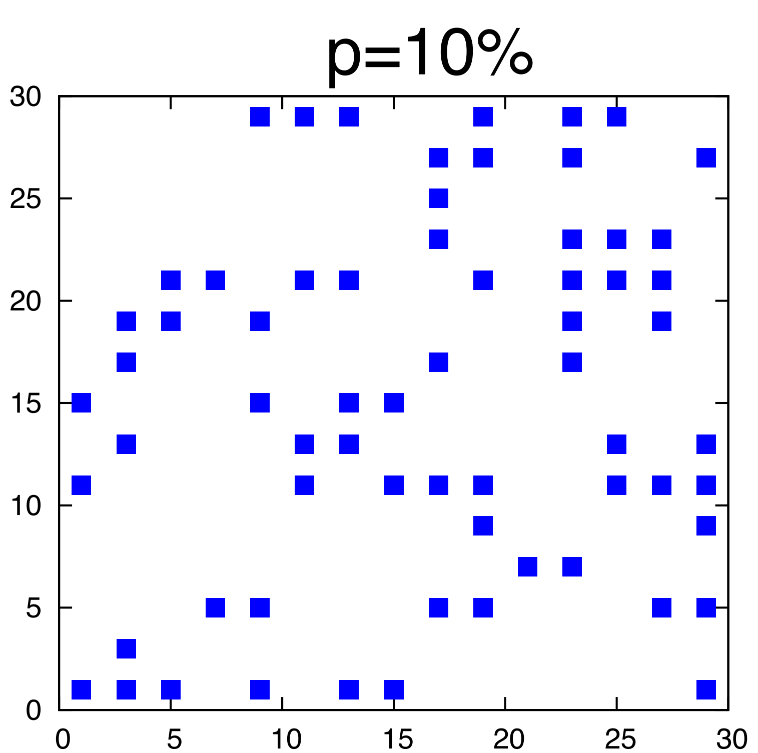


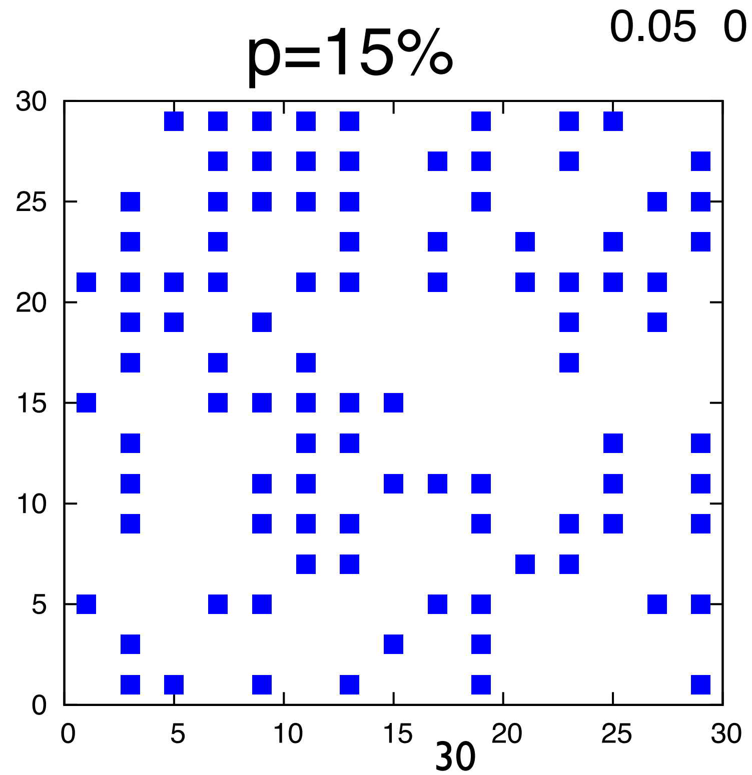
[Dennis et al 02]

Topological error correction









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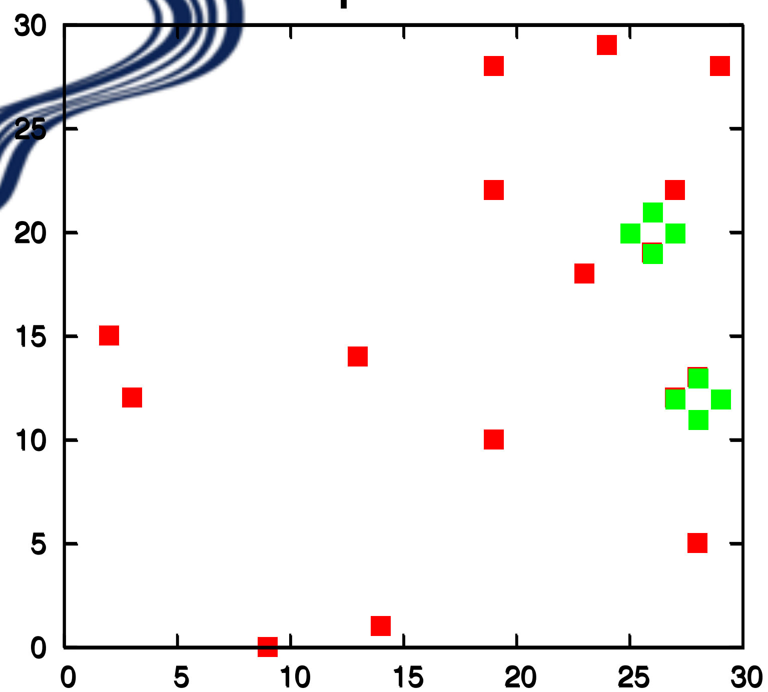

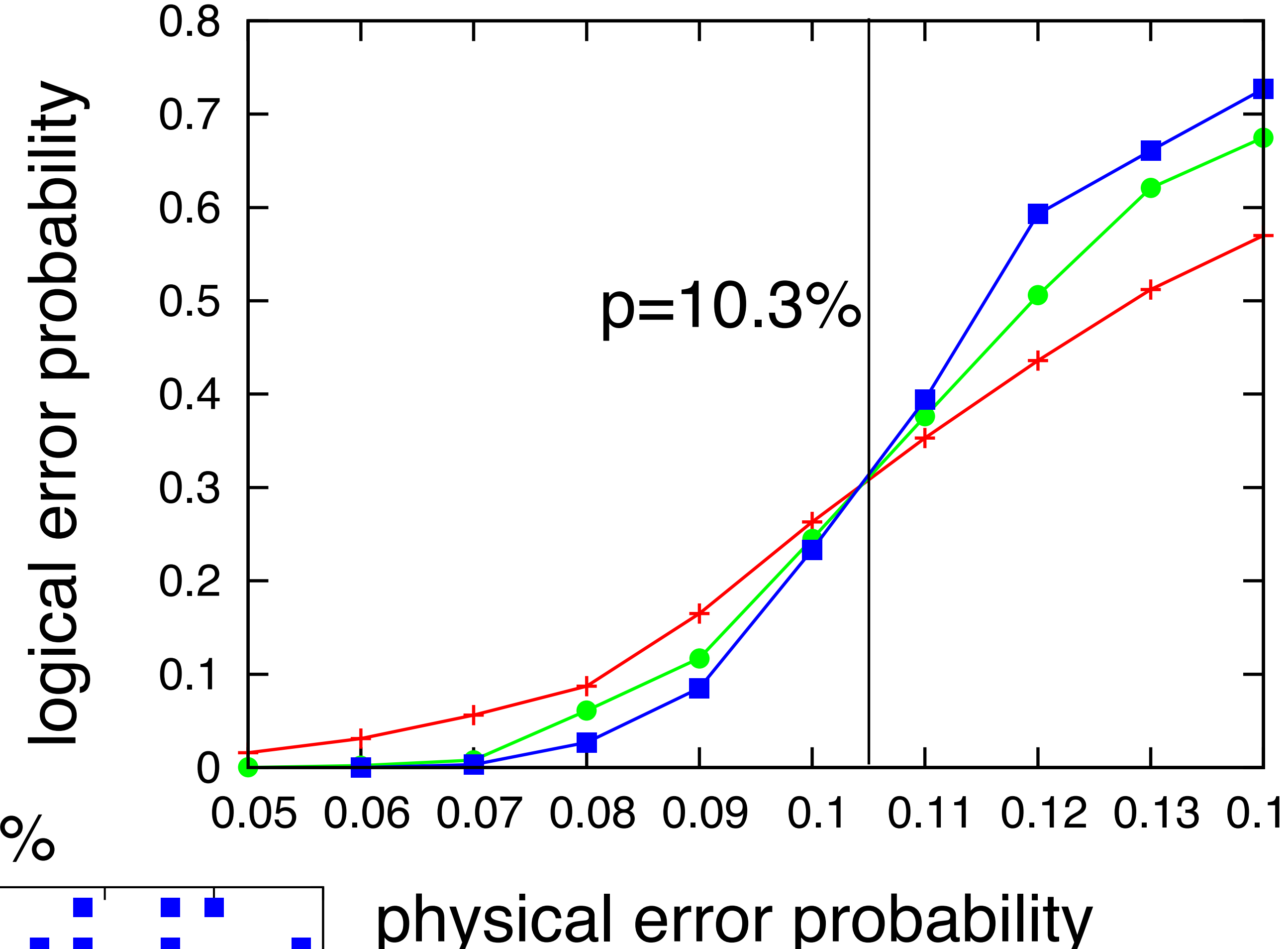

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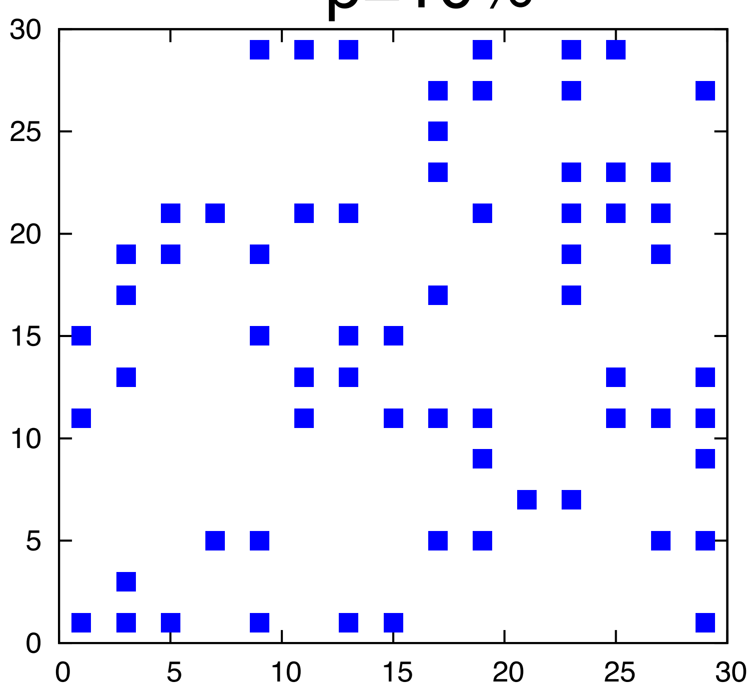

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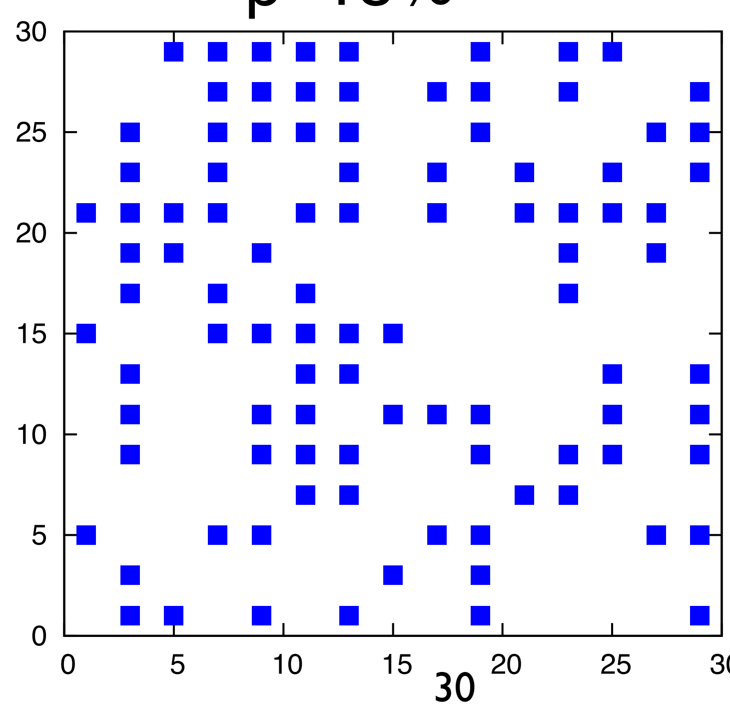

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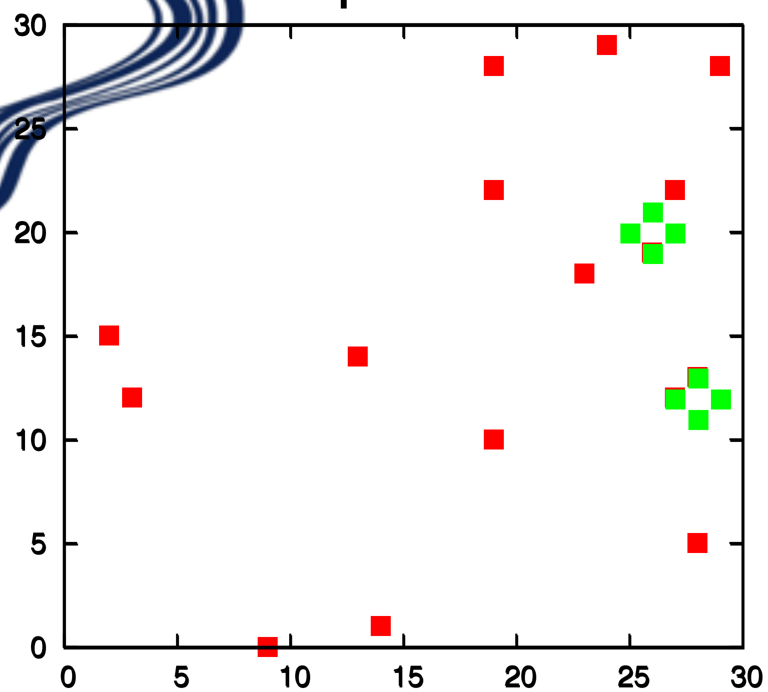
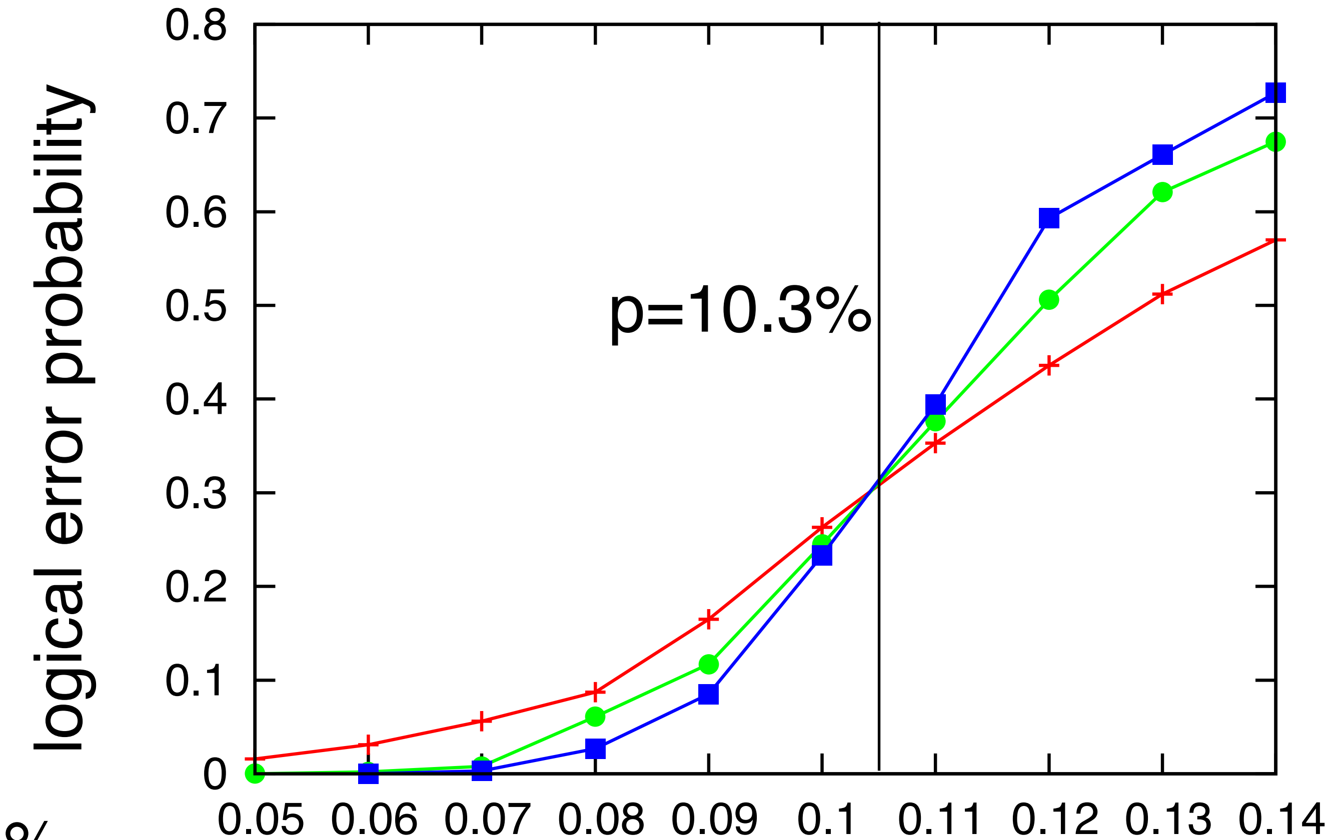
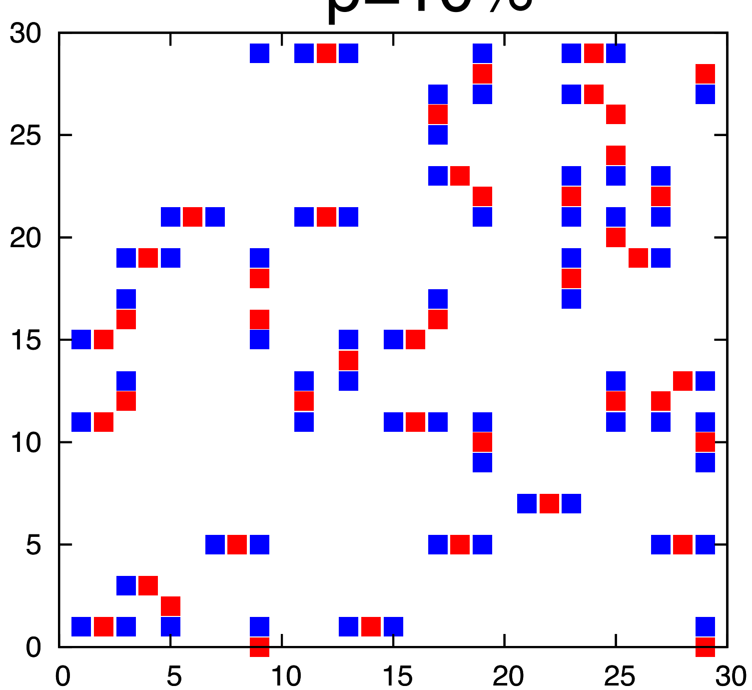




physical error probability

p=10%



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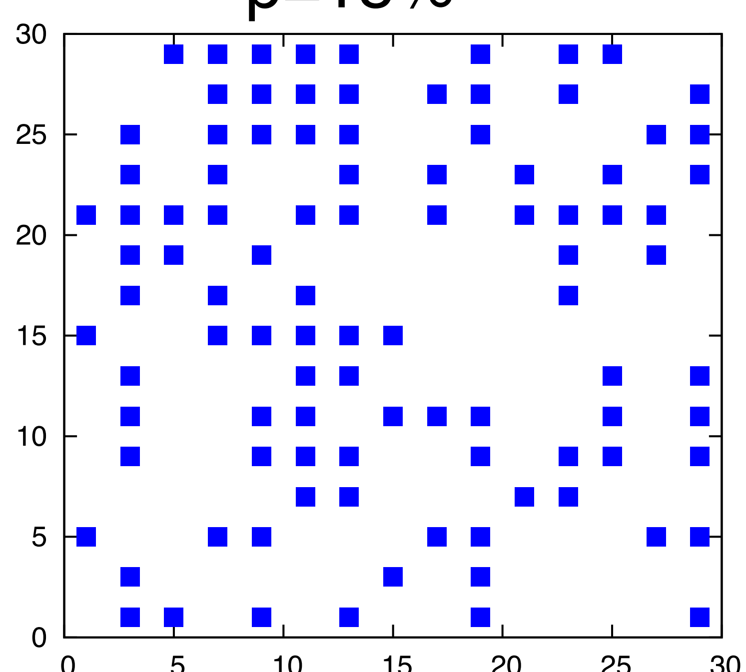

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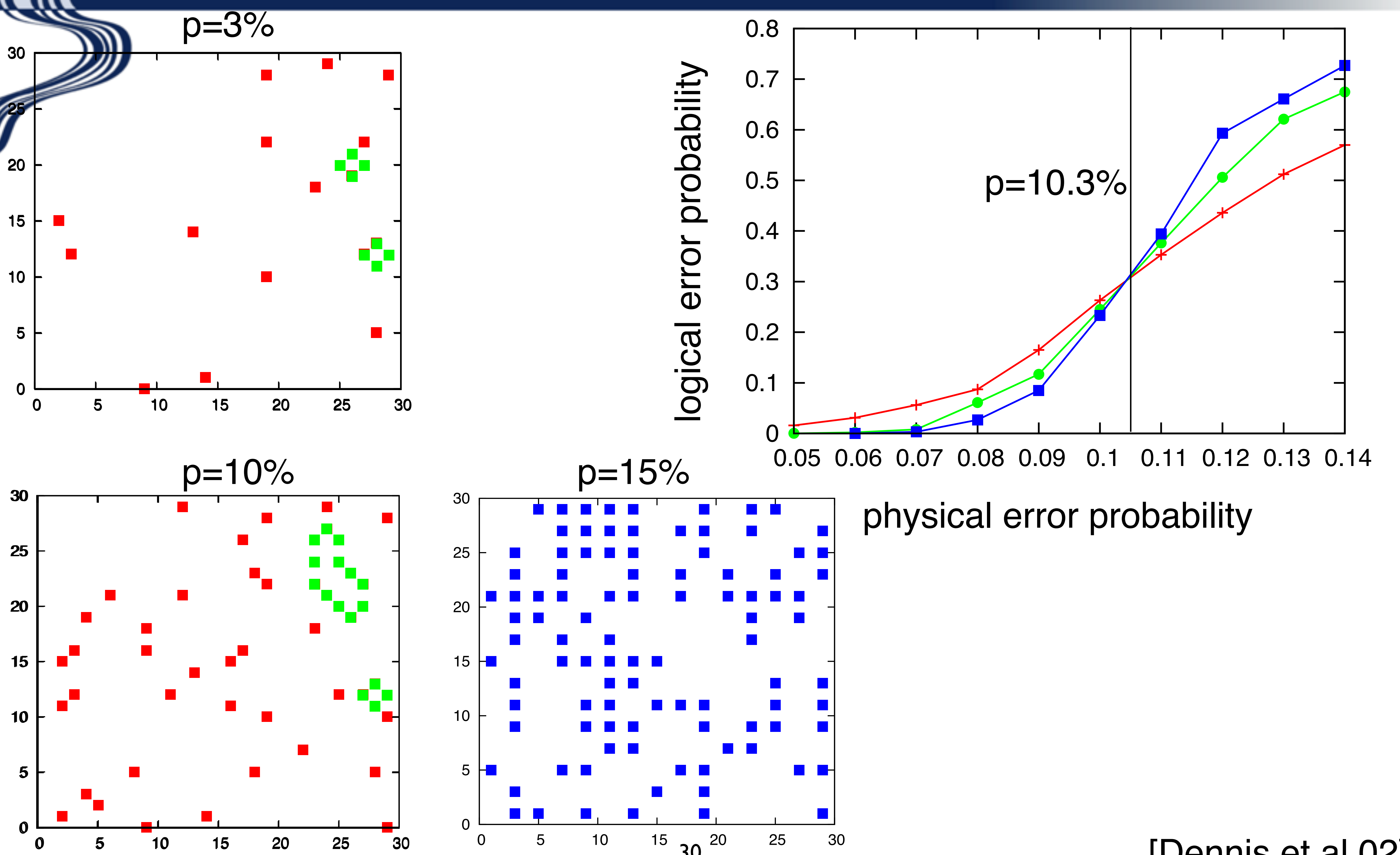
[Dennis et al 02]

Topological error correction



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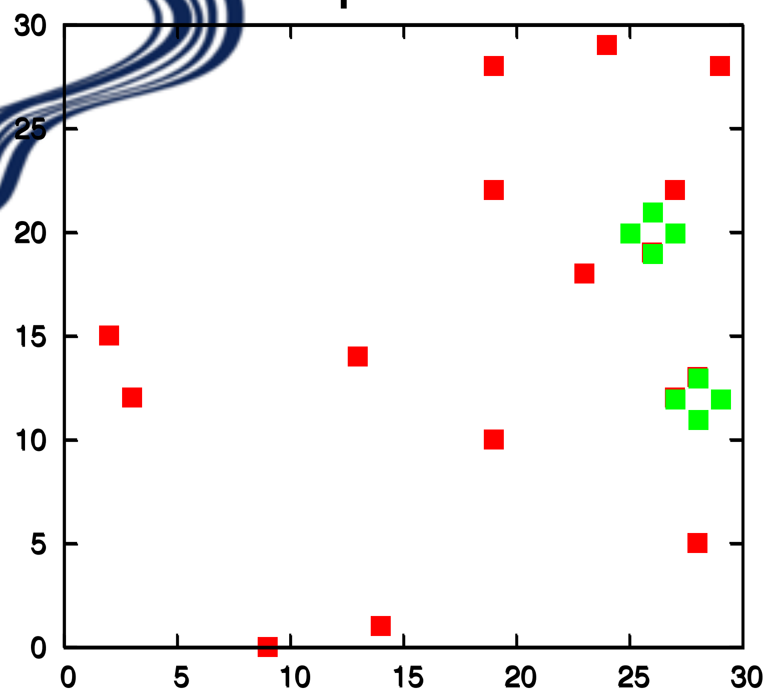

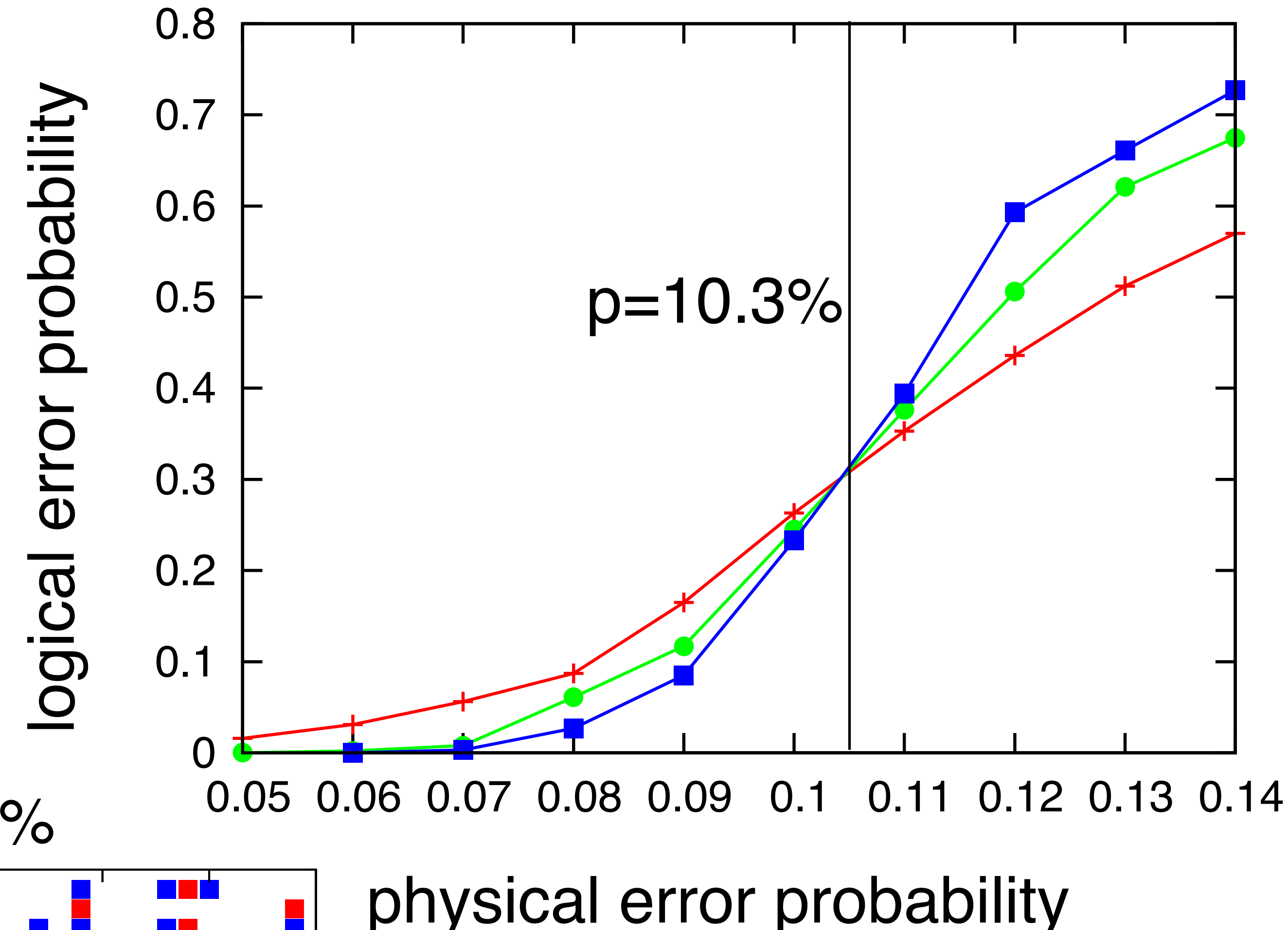

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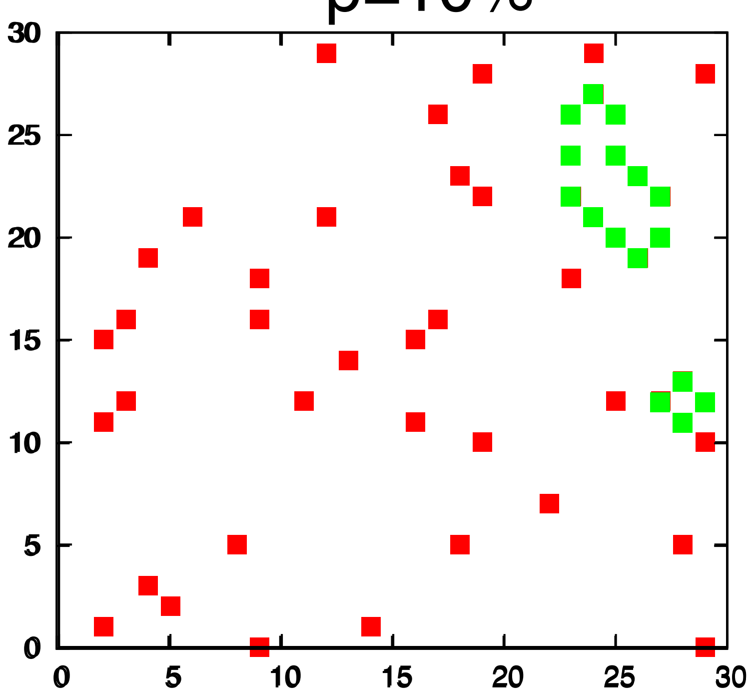

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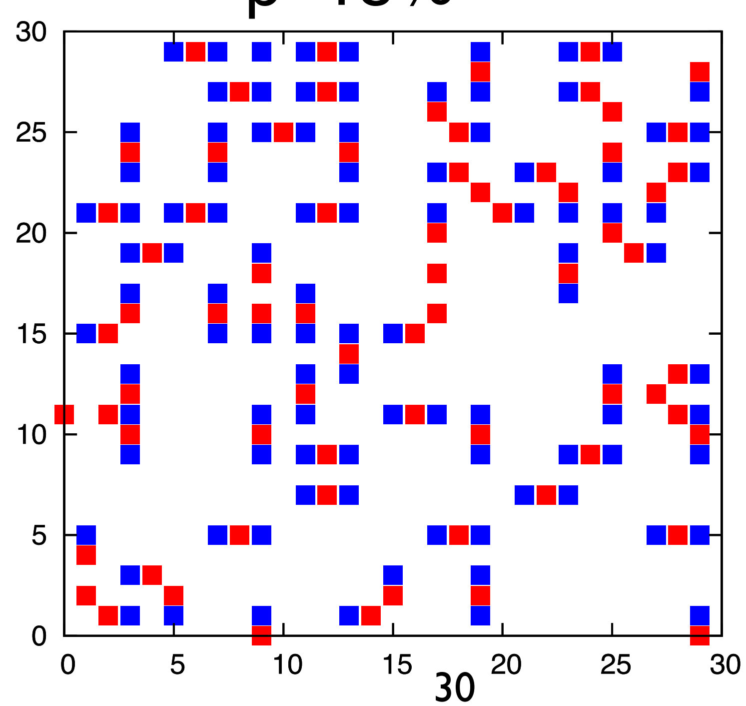

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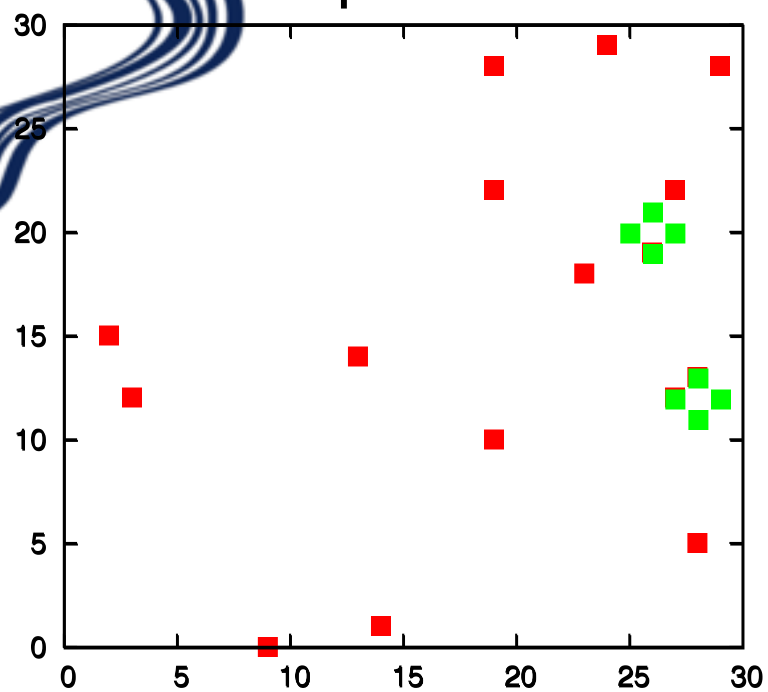

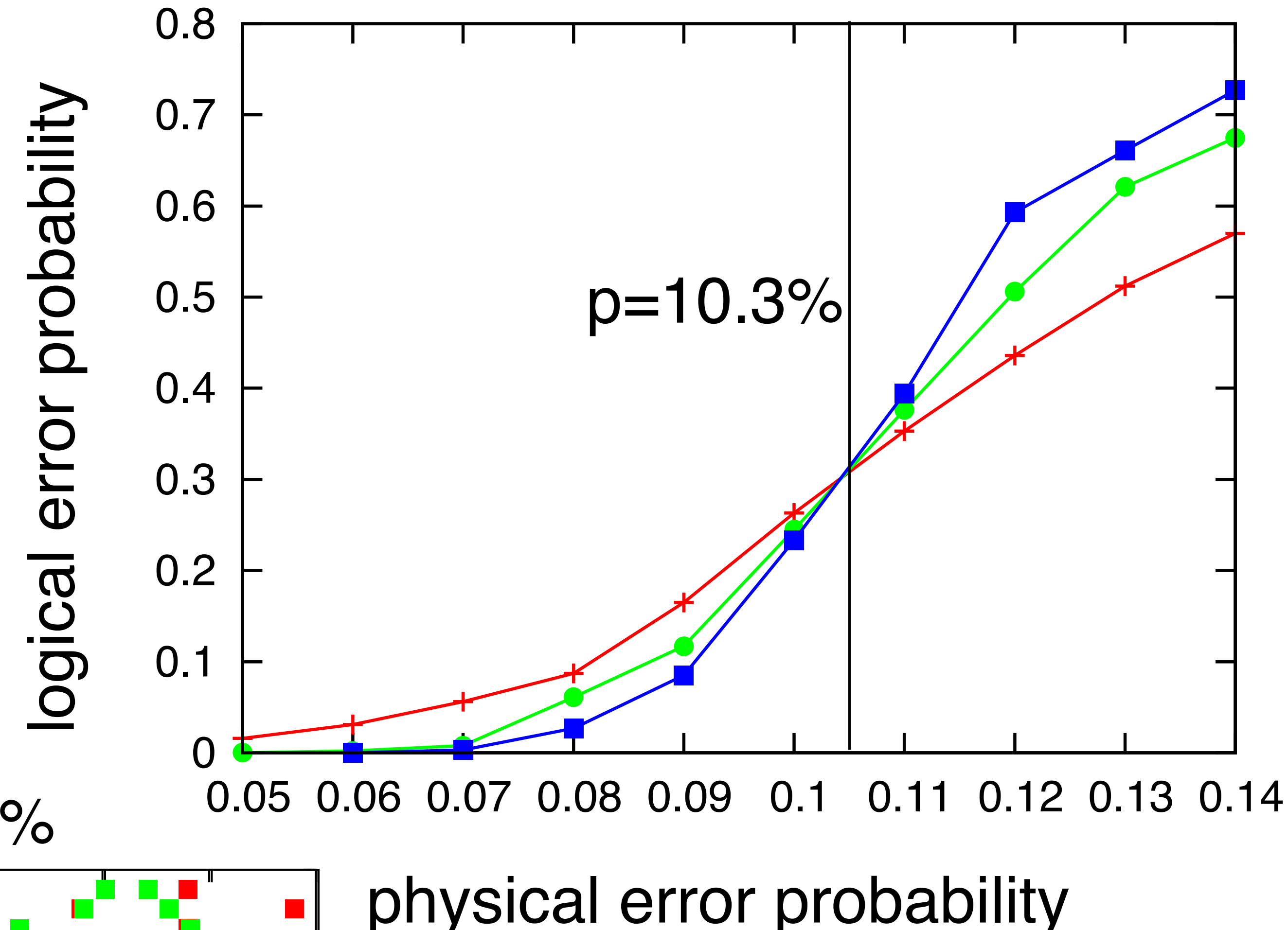

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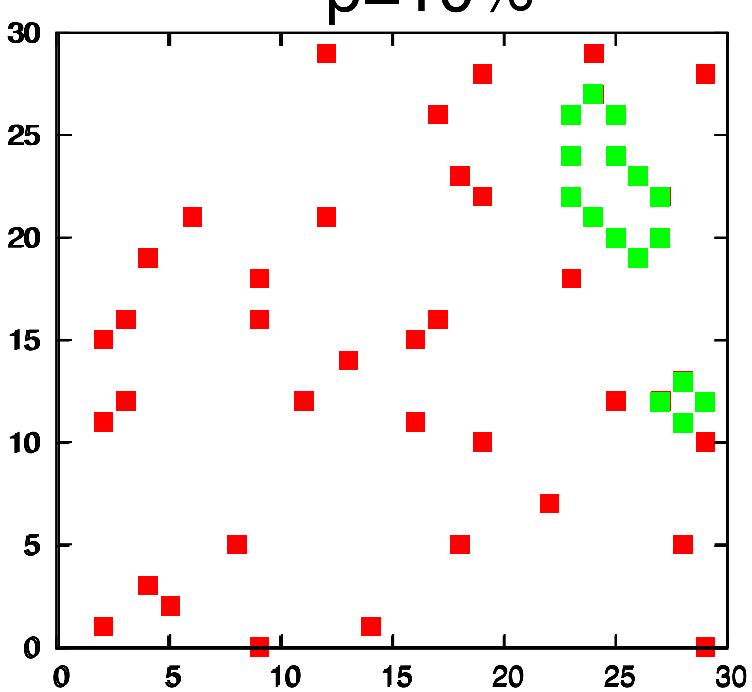

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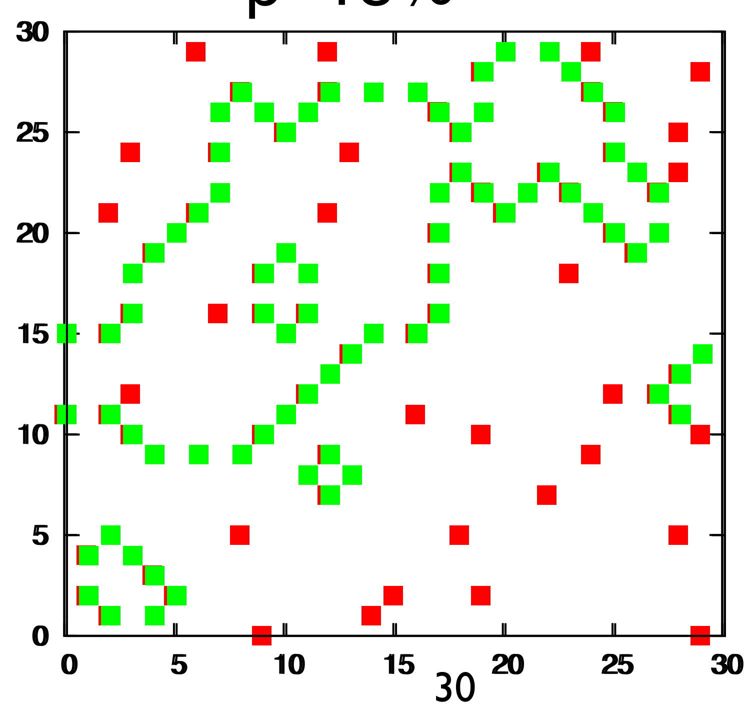

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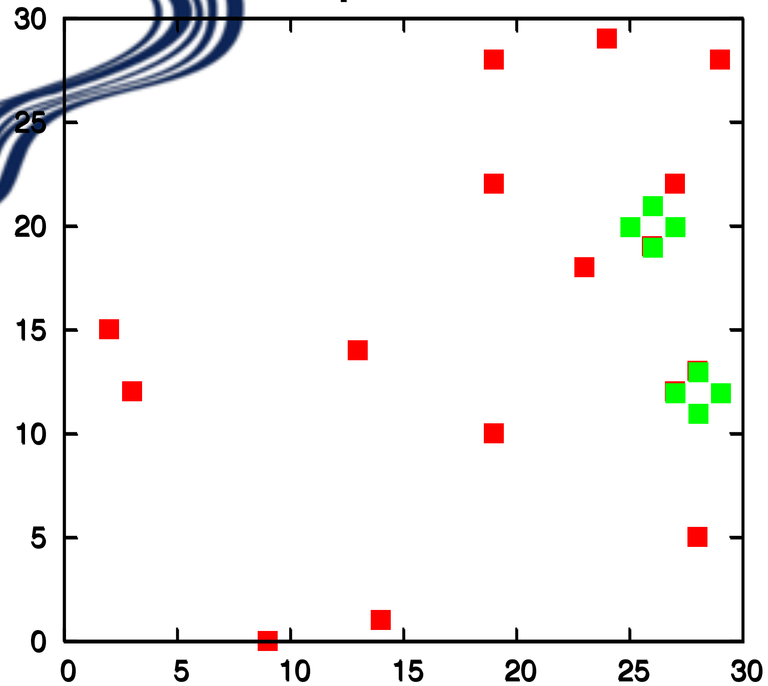

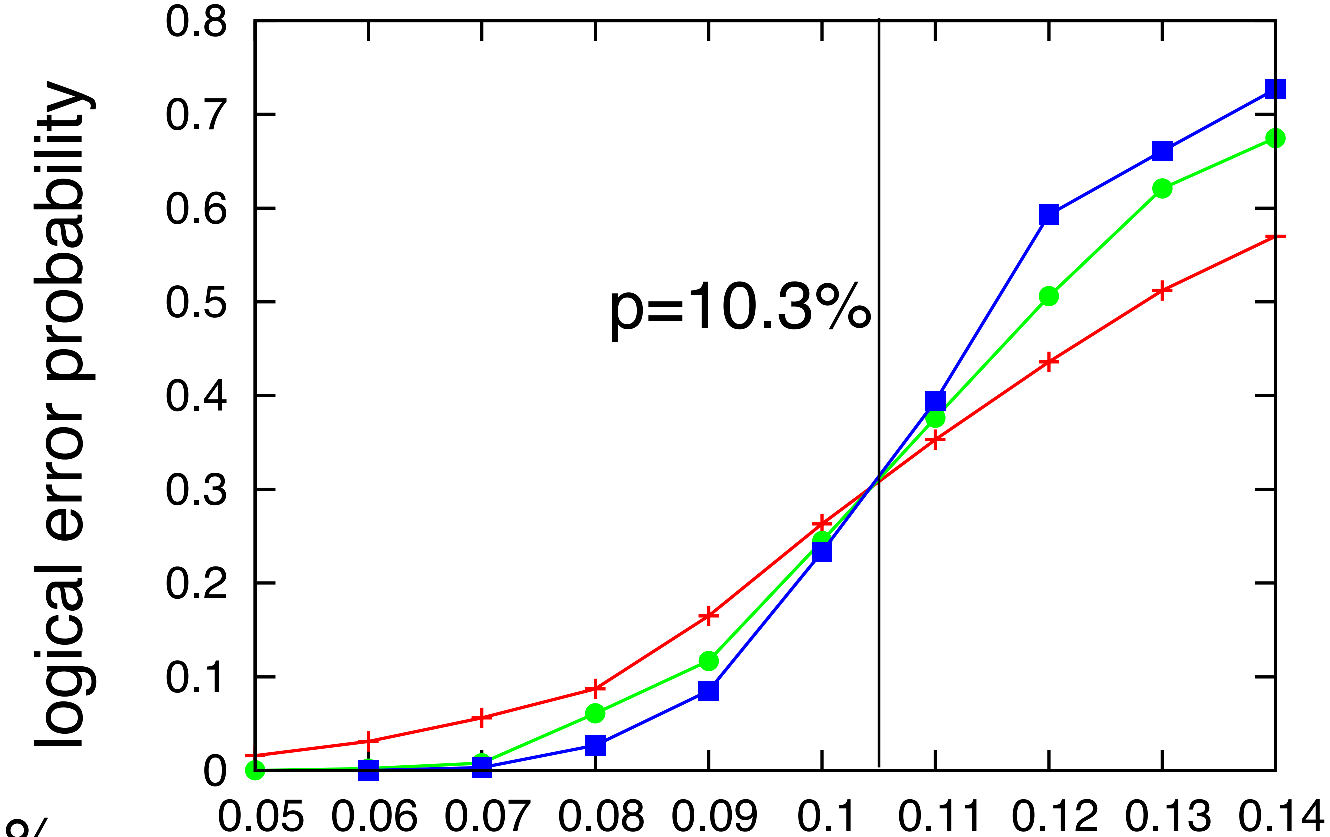

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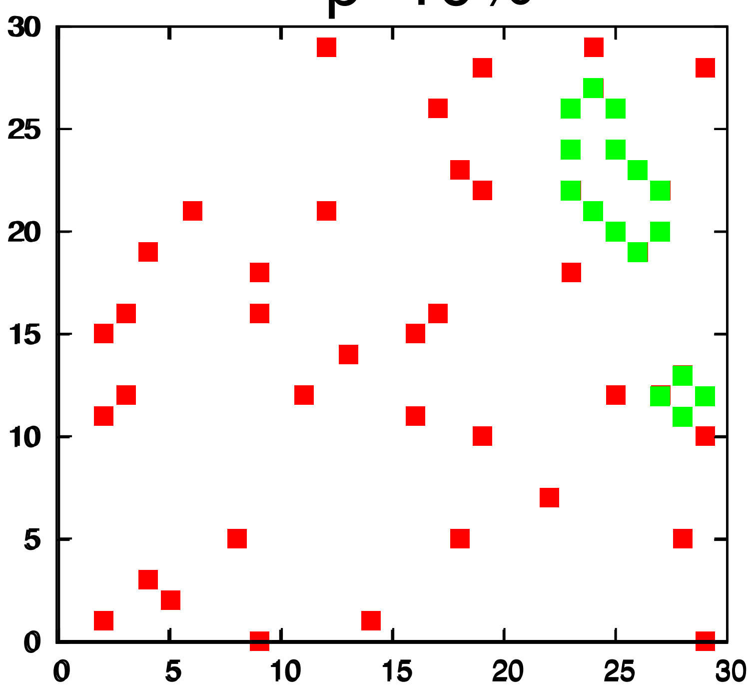

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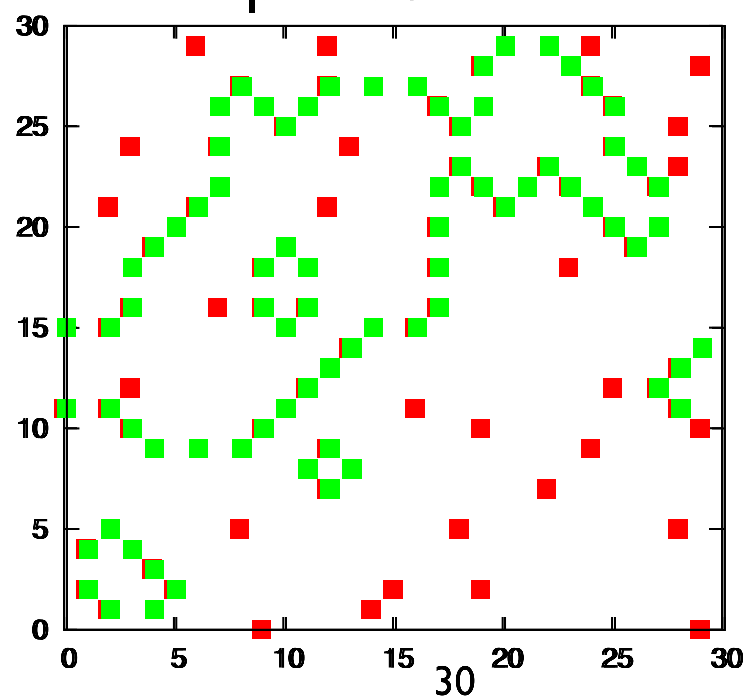

Is it related to some sort of phase transition?

Yes!

[Dennis et al 02]

Error correction and Spin glass model

Optimal error correction = maximizing posterior probability:

$$\arg\max_{L} p(L|S)$$

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$$\propto \mathcal{Z}_{\text{Ising}}(\{J_{ij}\})$$

partition function of random-bond Ising model
[van den Nest-Dur-Briegel 07]

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[van den Nest-Dur-Briegel 07]

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product state

toric code state = superposition of loops

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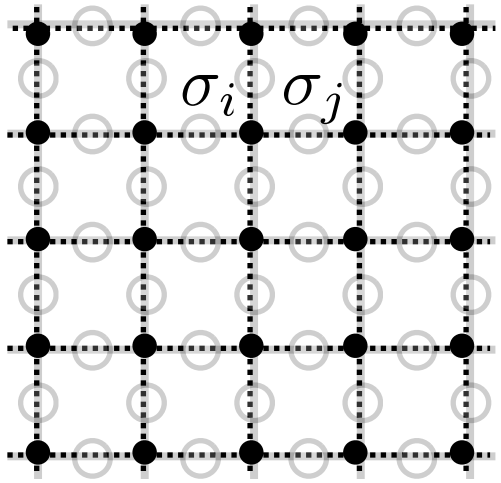

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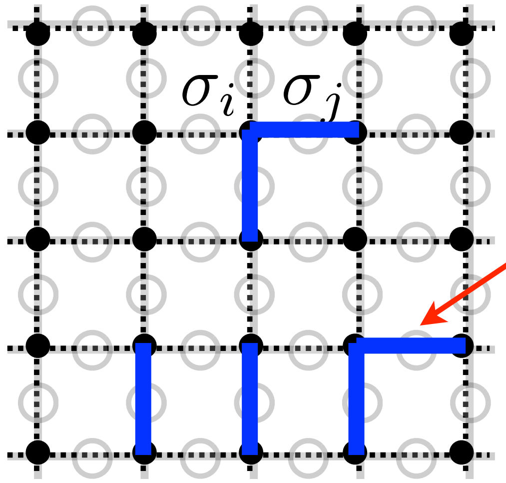

errors =
anti-ferro bonds

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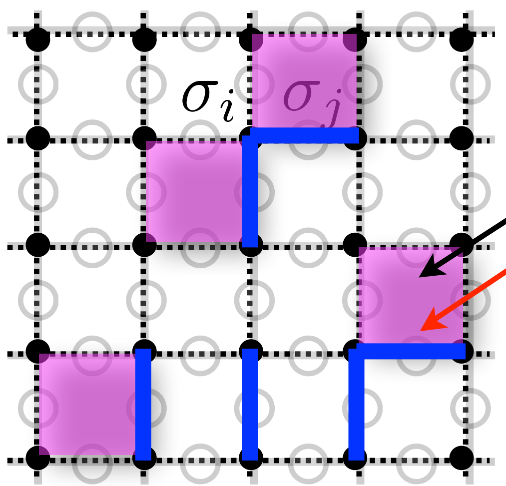

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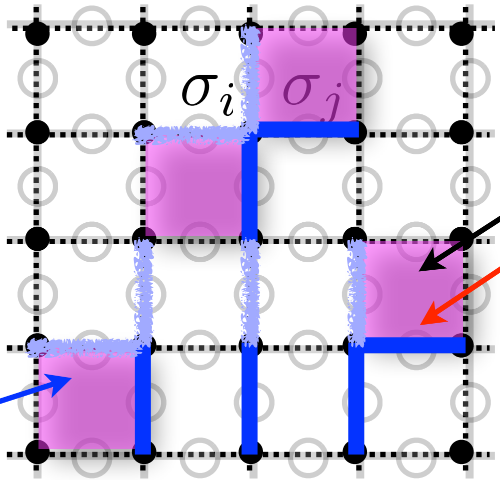

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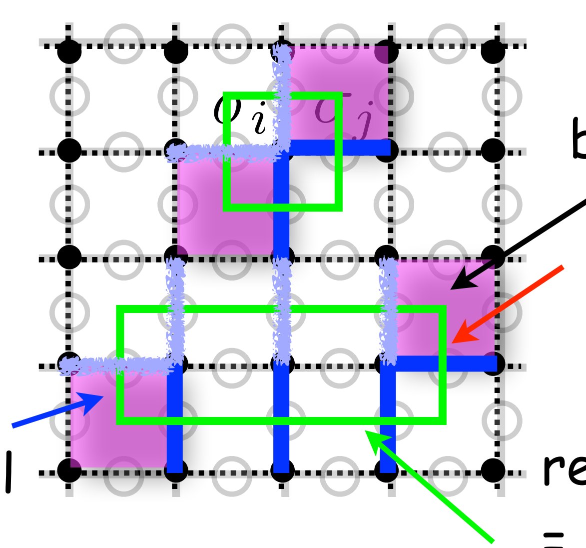

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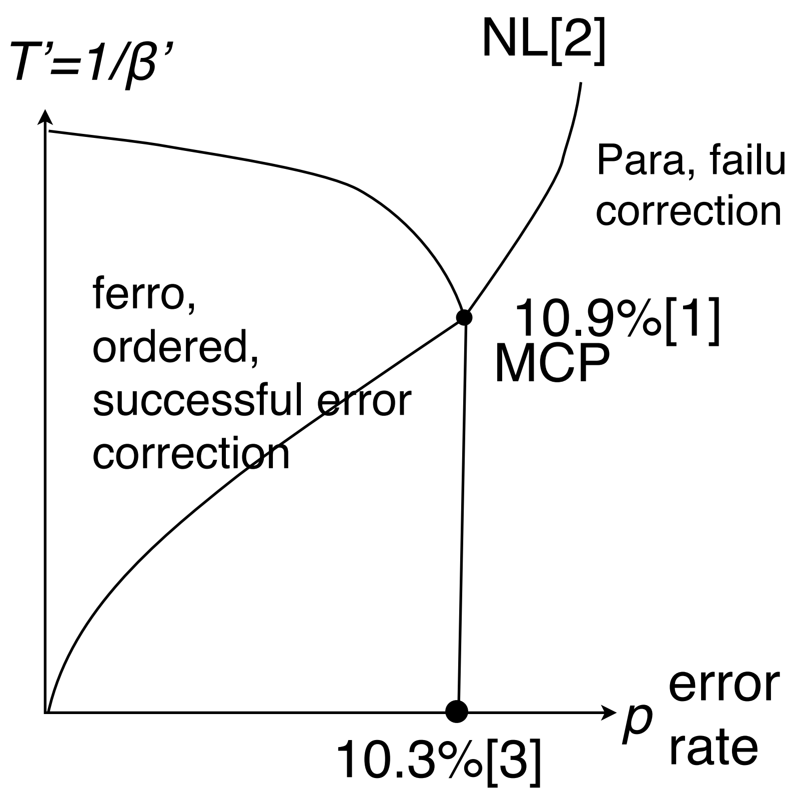
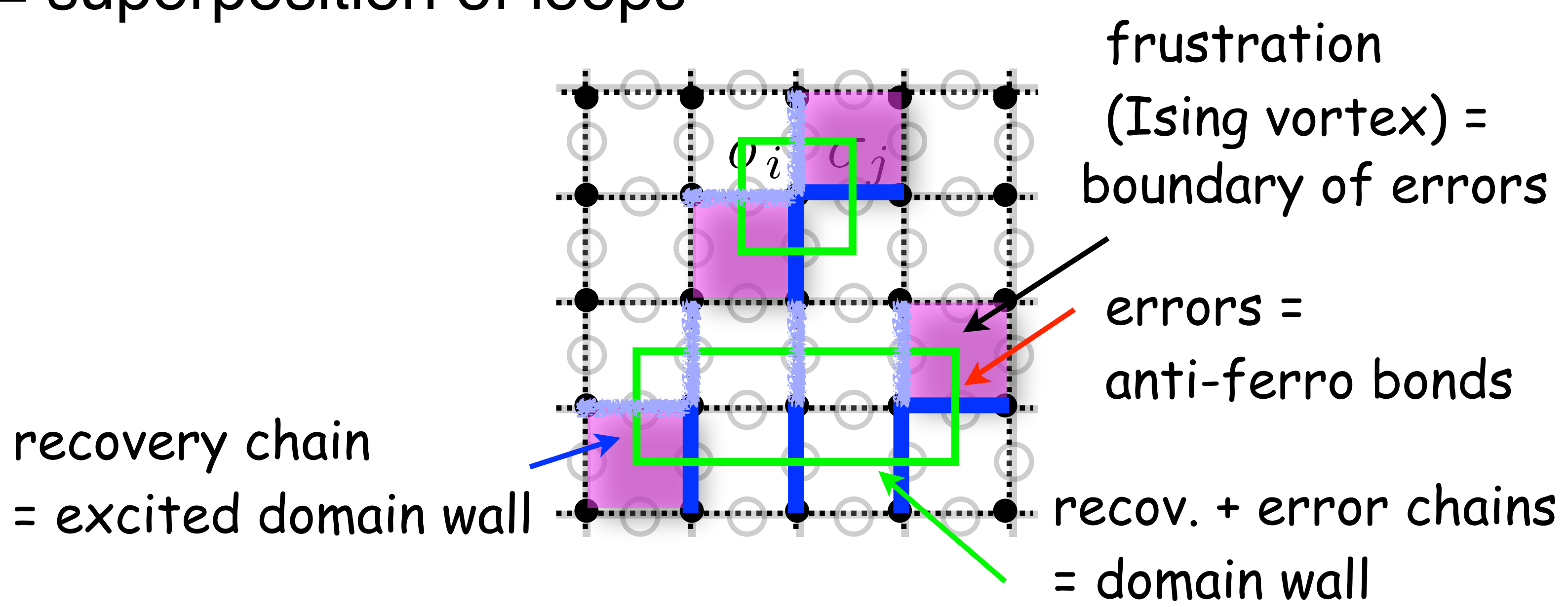


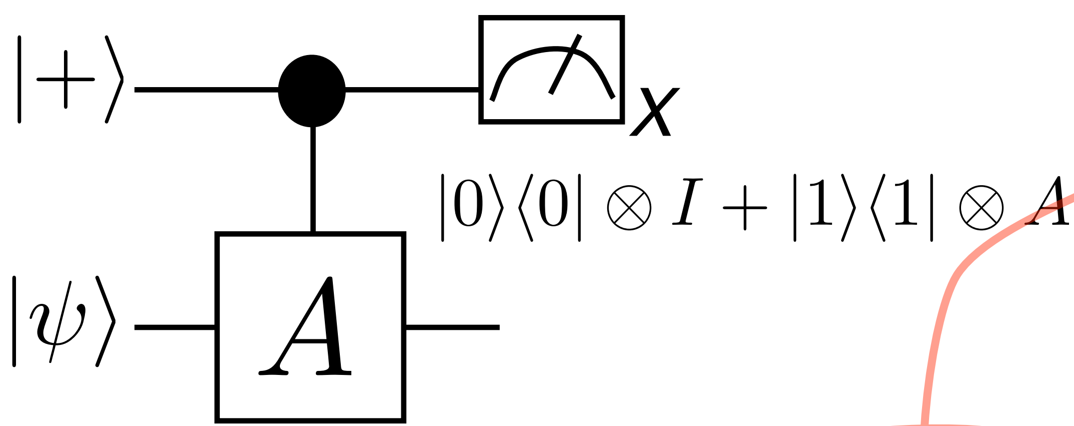


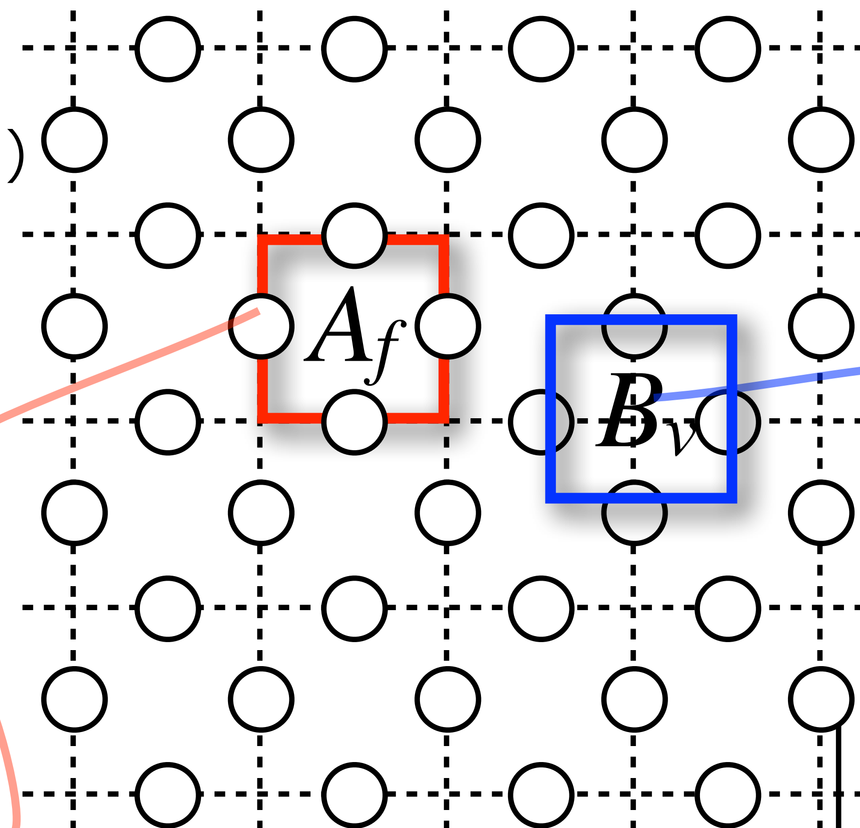
[1]H. Nishimori, Prog. Theor. Phys. **66**, 1169 (1981).
[2]M. Ohzeki, Phys. Rev. E **79**, 021129 (2009).
[3]C. Wang, J. Harrington, and J. Preskill, Ann. of Phys., **303**, 31 (2003).

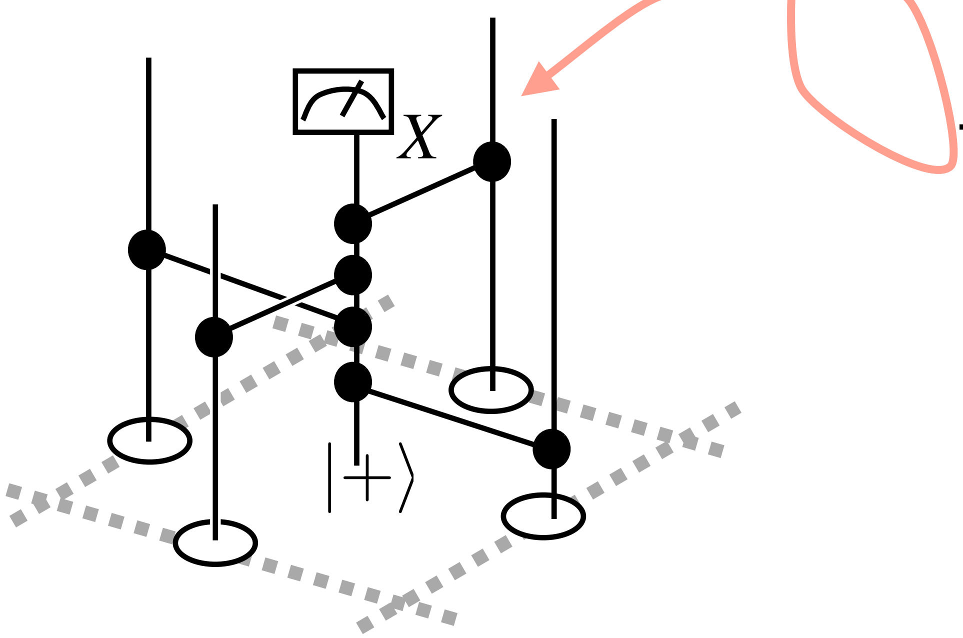
Syndrome measurement

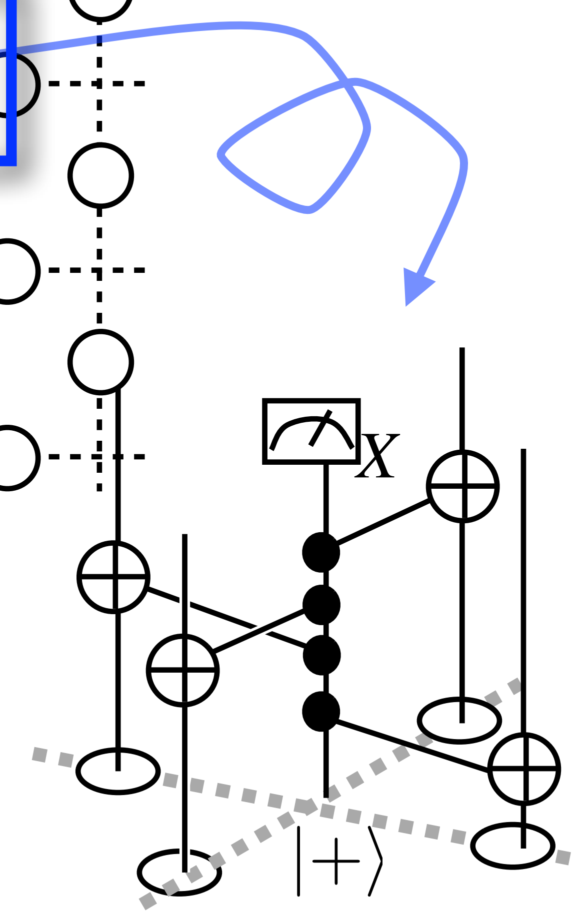
Measure the eigenvalues of the stabilizer operators.

Projective measurement for
an operator A (hermitian & eigenvalues ±1)









Topological quantum error correction & computation can be
implemented by using only nearest-neighbor gates in 2D.

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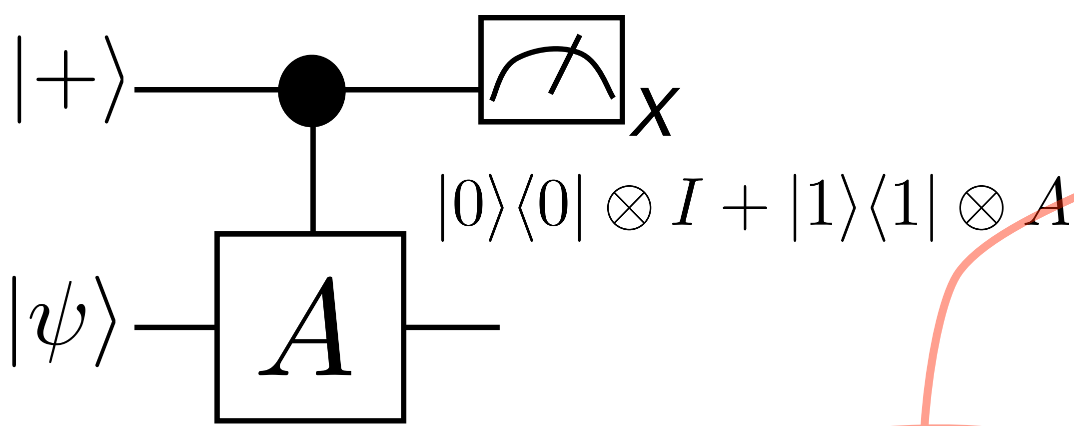
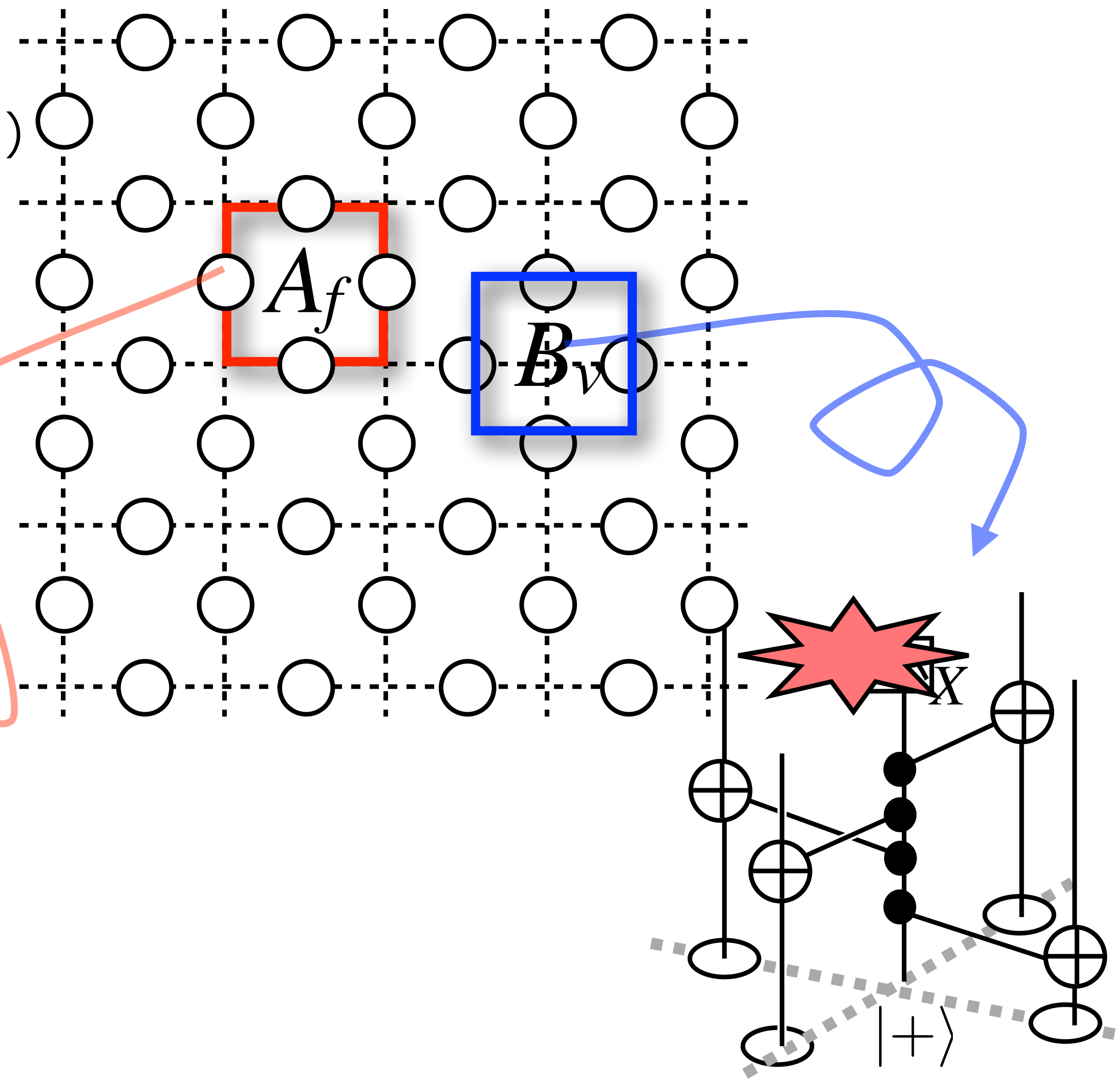
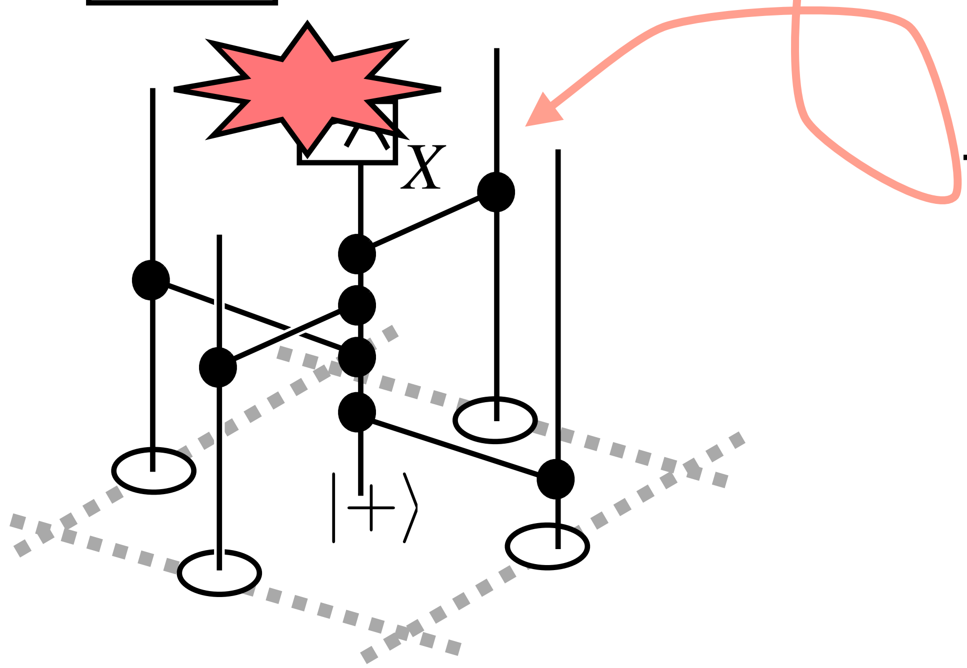
an operator A (hermitian & eigenvalues ±1)







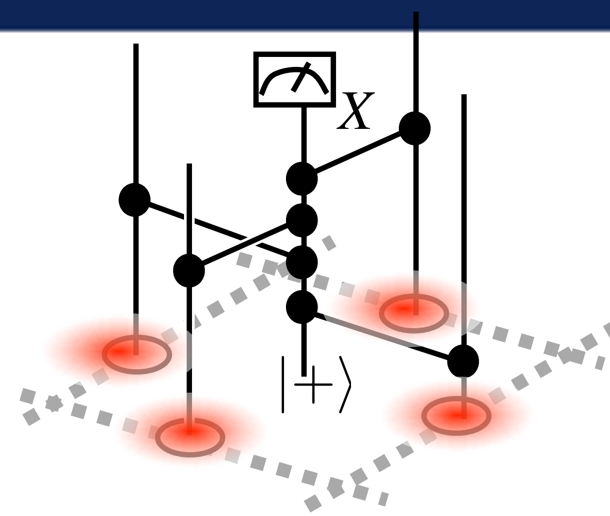
Topological quantum error correction & computation can be
implemented by using only nearest-neighbor gates in 2D.

Noise models and topological threshold values

Code performance noise (radom-bond Ising**):**

Independent X and Z errors with perfect syndrome measurements.

[10.3-10.9%]



Dennis *et al.*,
J. Math. Phys. **49**, 4452 (2002).
M. Ohzeki,
Phys. Rev. E **79** 021129 (2009).

Noise models and topological threshold values

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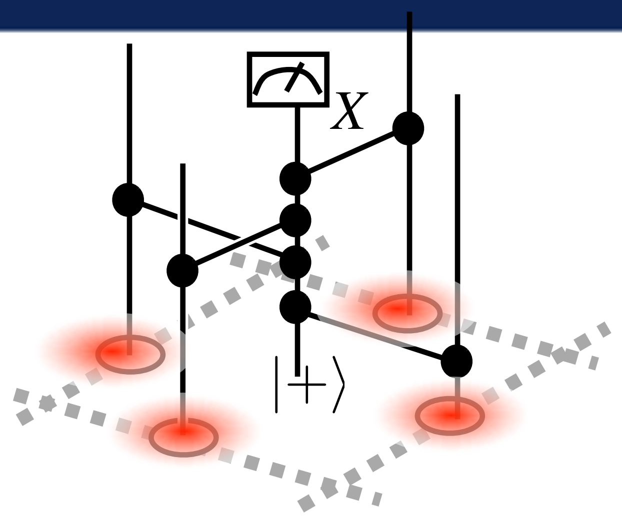

Dennis *et al.*,
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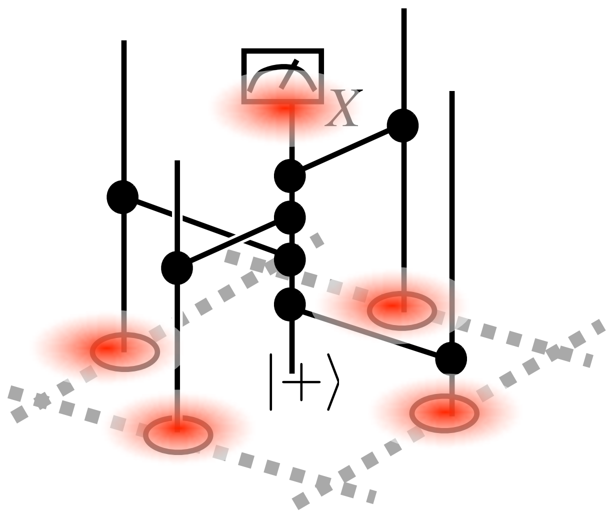
M. Ohzeki,
Phys. Rev. E **79** 021129 (2009).

Phenomenological noise model (random-plaqette Z2 gauge**):**

Independent X and Z errors with noisy syndrome measurements.

[2.9-3.3%]



Wang-Harrington-Preskill,
Ann. Phys. **303**, 31 (2003).
Ohno *et al.*,
Nuc. Phys. B **697**, 462 (2004).

Noise models and topological threshold values

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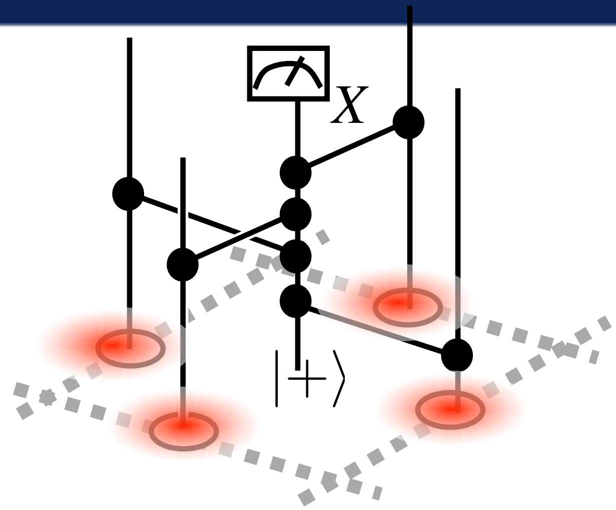

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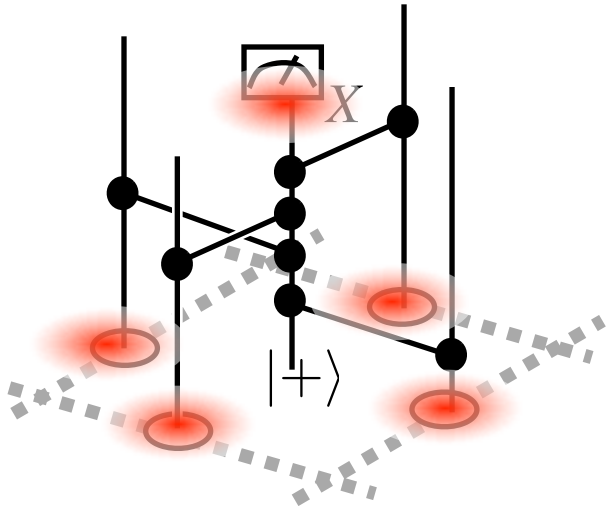
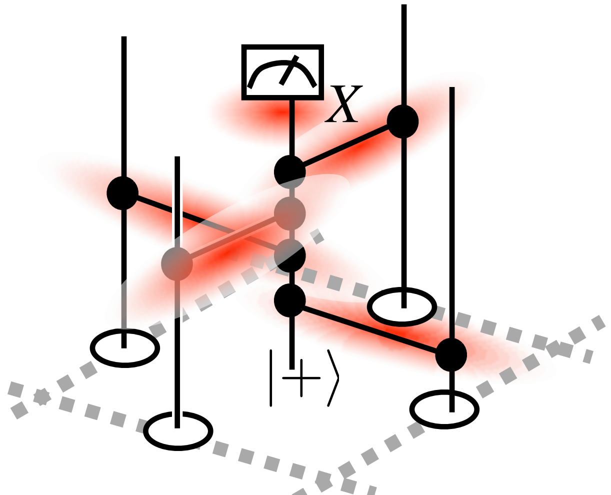


Wang-Harrington-Preskill, Ann. Phys. **303**, 31 (2003).
Ohno *et al*., Nuc. Phys. B **697**, 462 (2004).

Circuit noise model:

Errors are introduced by each elementary gate.

[0.75%]



Raussendorf-Harrington-Goyal, NJP **9**, 199 (2007).
Raussendorf-Harrington-Goyal, Ann. Phys. **321**, 2242 (2006).

SUPERCONDUCTING QUBITS

Solving a wonderful problem

Superconducting qubits are used to demonstrate features of quantum fault tolerance, making an important step towards the realization of a practical quantum machine.

Simon Benjamin and Julian Kelly

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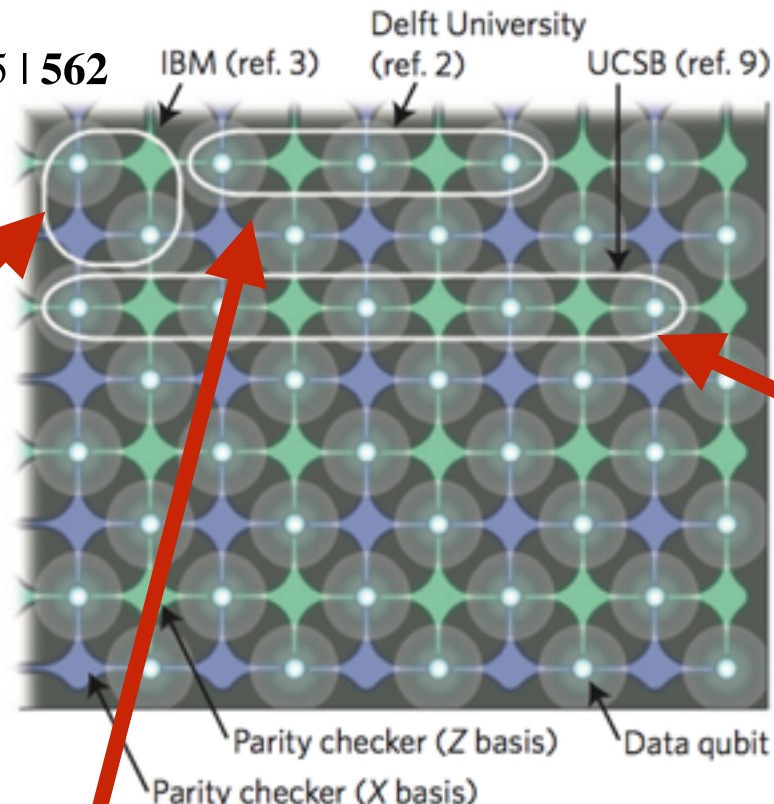
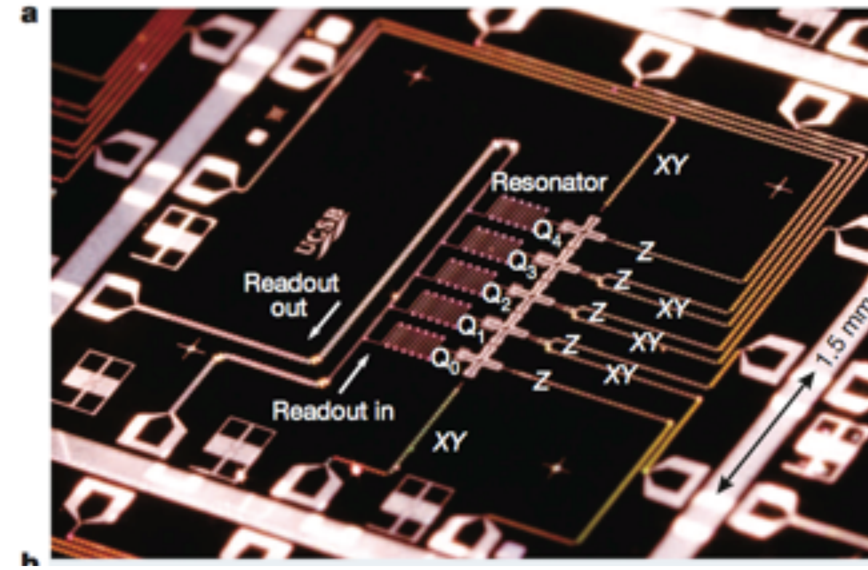
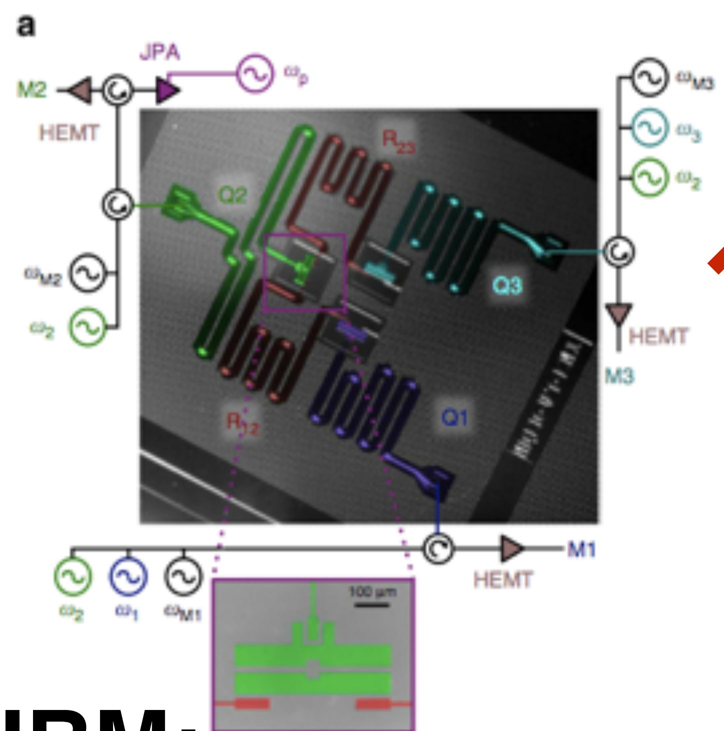
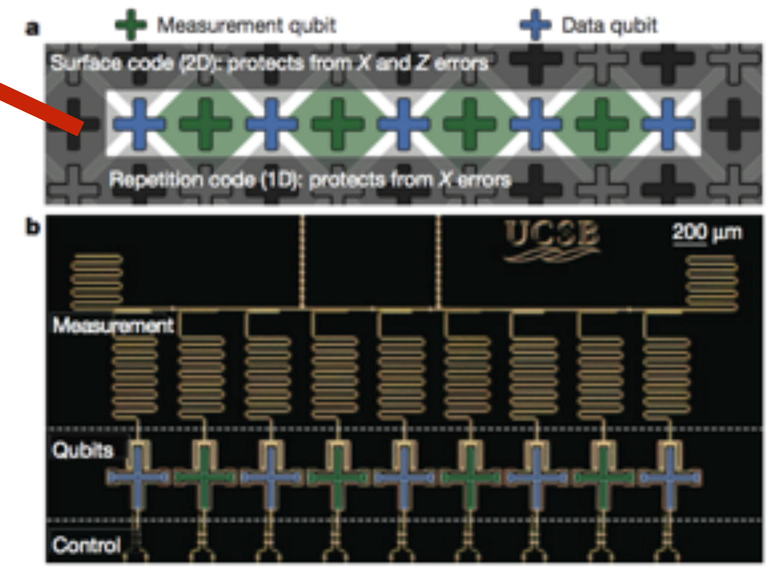








IBM:

Chow et al., Nat. Comm. **5** 4015 (2015)

UCSB+Google:

Kelly et al., Nature **519,** 66 (2015)
Barends et al., Nature **508**, 500 (2014)

[fidelities]

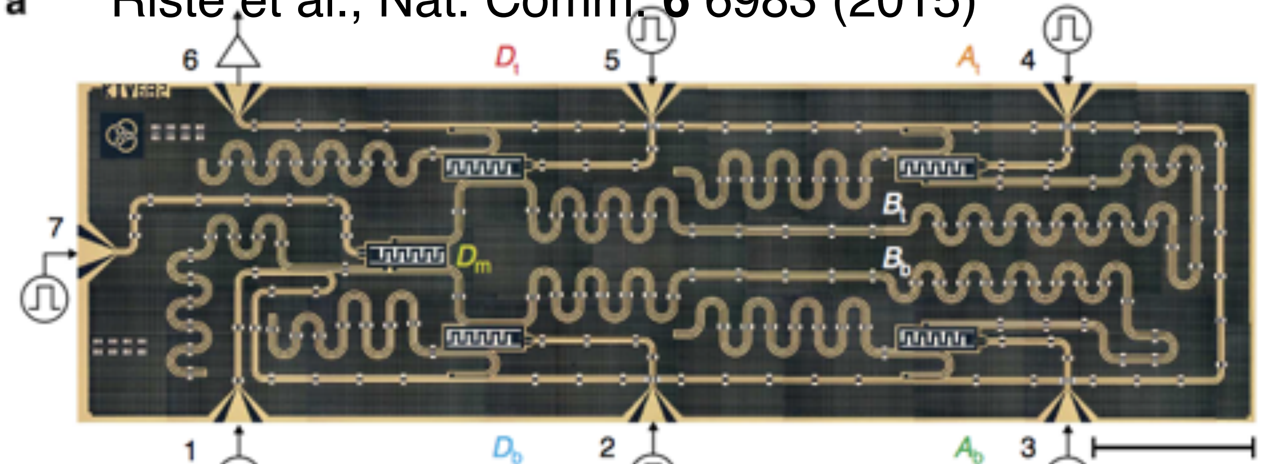
single-qubit gate: 99.92%

two-qubit gate: 99.4%

measurement: 99%

Delft+Intel:

Riste et al., Nat. Comm. **6** 6983 (2015)



Summary

Topologically ordered system in 2D

→Kitaev's toric code, topological property, logical operators

The relations between quantum error correction and topological order/ spin glass model

How topological quantum error correction works